1939

The balanced design of reinforced, rectangular concrete beams, M. S. Thesis, Lehigh University, 1939

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THE BALANCED DESIGN OF REINFORCED, RECTANGULAR CONCRETE BEAMS

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A THESIS

Presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Master of Science

Lehigh University

1939

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

Professor in Charge

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ACKNOWLEDGMENT

The test specimens were fabricated and the tests were made in the Fritz Engineering Laboratory at Lehigh University. Acknowledgment is made to the Concrete Reinforcing Steel Institute and to the Carnegie-Illinois Steel Corporation for furnishing the reinforcing steel; to the Lehigh Portland Cement Company for furnishing the cement; to the Fritz Engineering Laboratory for the remaining materials; and to Inge Lyse, formerly Research Professor of Engineering Materials and now Professor of Reinforced Concrete and Solid Bridges at the Norges Tekniske Høiskole at Trondheim, Norway. Dr. Bruce Johnston, Assistant Director of Fritz Engineering Laboratory; Mr. Howard Godfrey, Engineer of Tests at Fritz Engineering Laboratory, and the other members of the Fritz Laboratory Research Staff are also due acknowledgment for their assistance in preparation and testing of the specimens as well as for their continued interest and suggestions.

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I - SYNOPSIS

In the design of reinforced concrete, it is generally agreed that the steel and the concrete should be so proportioned that each material will be utilized to the fullest extent. When a reinforced concrete section is so constructed it is said to have a "balanced design."

The investigation described in the following report was carried out in the Fritz Engineering Laboratory of Lehigh University in order to find, by means of experimental tests, the relation between steel and concrete for "balanced design".

The program consisted of making and testing one hundred and ten rectangular concrete beams. The variables included the strength of concrete, the percentage of steel, and the size of steel. All of the beams were of the same dimensions except for six beams of varying depth which were designed as a study of this effect. Three 6 by 12-in. control cylinders were made for each set of nine beams.

The physical properties of the reinforcing steel were determined by A.S.T.M. standard procedure.

The load was applied to the beams by an apparatus specially designed to give the effect of dead weight loading.

In addition to the tests necessary for studying "balanced design" tests were made to determine the effect of vibration on control cylinders, the effect of size of control cylinder, the effectiveness of compression reinforcing, the effect of depth of a beam, and other related ideas.

II - INTRODUCTION

Purpose and Scope - This thesis was designed to determine experimentally the maximum amount of steel that can be fully utilized in a reinforced concrete beam. This question, practical in nature, should indicate the most economical design for flexure in reinforced concrete.

The term "balanced design" can be found in nearly every text book on reinforced concrete. By "balanced design" one means the utilization of the full strength of concrete and the yield-point strength of the steel. Very little data were found on this question, and the writer knows of no program which has been designed primarily for this purpose. Theoretically it is possible to compute the percentage of steel for "balanced design". Since reinforced concrete does not behave as an elastic material and is non-homogeneous a practical analysis must necessarily be based upon various empirical assumptions.

In this investigation four strengths of concrete and various cross-sectional areas of steel were employed to accomplish the purpose stated above. The concrete cylinders; 6 by 12 in. vibrated, 6 by 12 in. rodded, and 3 by 6 in. rodded; were made to study the effect of vibration and cylinder size upon the compression strength.

Various methods of design were used in analyzing the results of these tests in order to compare the relative merits of each.

III - TEST PROGRAM AND PROCEDURE

A. General Statement - In this study of the relation between concrete and steel it was necessary to employ various strengths of concrete and many different areas of steel.

The beams were made in twelve groups, each consisting of nine 5 by 6 by 50-in. beams. Three vibrated 6 by 12 in. compression cylinders were made in order to check the compression strength of the concrete. For each cement-water ratio three 6 by 12-in. standard rodded cylinders and three 3 by 6-in. standard rodded cylinders were made in addition to the 6 by 12-in. vibrated specimens.

The testing of the beams consisted of reading the deflection at various load increments and noting the yielding of the steel, the progression of cracks and the ultimate load.

B. Concrete - The concrete was designed after trial batches were made to determine the amount of water for desirable workability. It was found that 325 pounds of water should be used per cubic yard of concrete for a slump of one inch as determined by the slump cone method. Two pilot tests were made with the concrete from the trial batches. The concrete strength was varied by using four different cement-water ratios. These ratios were: 1.00, 1.25, 1.50, and 1.75.

The concrete mixes used in pouring the beams are given in Table I. The coarse aggregate consisted of crushed limestone and was limited to a three-eighths inch maximum size so that it would pass between the reinforcing bars. The remaining materials consisted of a fine sand from northern New Jersey, tap water, and Lehigh Portland Cement.

The concrete was mixed in two-and one-quarter cubic foot batches in a "stirrer" type mixer for three minutes. The concrete was then poured into oiled steel forms and vibrated carefully with an electric vibrator as shown in Fig. 1. At the age of one day the specimens were removed from the forms and placed in the moist room until they reached the age of 28 days. The temperature of the moist room was 70°F and the humidity was 100 per cent.

At least three 6 by 12-in. vibrated control cylinders were made with each group of nine beams. These cylinders were made by half filling the cylinder mould, then vibrating the concrete for twenty seconds with the electric vibrator. The remaining half of the cylinder mould was then filled and the vibrator again applied for twenty seconds, after which the top was carefully "struck off".

For each cement-water ratio, in addition to the vibrated 6 by 12-in. cylinders, three 6 by 12-in. and three 3 by 6-in. cylinders were made by the A.S.T.M. standard method of hand rodding. All the cylinders were capped with neat cement paste and tested at the age of 28 days.

Three 6 by 12-in. vibrated concrete cylinders having a cement-water ratio of 1.25 were tested to determine their modulus of elasticity in compression. This was done by fastening two steel collars to the cylinders eight inches apart. The collars were held to the cylinder by three set screws spaced 120 degrees apart. The deformation between the collars was read by four Ames dials reading to 1/10,000-in. The dials were spaced 90 degrees apart and equidistant from the cylinder. The compression cylinders were tested in a 300,000-lb. Olsen testing machine and readings were taken at 5000-lb. increments.

C. Steel - The physical properties of the steel, given in Table II, were determined by testing the reinforcing bars in a 50,000-lb. Riehle testing machine. The speed of testing was 0.05-in. per minute up to the yield point of the steel. The yield point was determined by the drop of the beam method; and the ultimate load by testing the bars to rupture. The initial diameter and diameter at fracture were read with a micrometer.

The steel bars used in each beam are given in Tables III to V inclusive. The bars were provided with a hook on each end to give additional bond. The bars were bent by a hand-operated bending machine. To keep the bars correctly and at the correct height above the bottom of the forms, bolsters of various dimensions were welded to the underside of the bars approximately seven or eight inches from the hooked end. The bar assemblies are shown in Fig. 2 to 6 inclusive.

Fig. 7 illustrates the design of the beams for the major part of the program. The arrangement of the bars necessary to produce a "balanced design" or an over design of steel often presented quite a problem. In some cases two layers of steel had to be used with a 3/8-in. spacer between them. Care was always taken to keep the center of gravity of all the bars one inch from the bottom and thus have an effective depth of five inches. In the case of the 1/4-in. size it was necessary to place several bars directly beside one another.

The stirrups, 3/8"ø deformed bars having a yield point as given in Table II were bent in a hand-operated stirrup bending machine. The number of stirrups used in each end of the beams is given in Tables III to V inclusive. This number was determined by Fig. 8 which was calculated using an allowable unit tensile stress of 30,000 p.s.i. in the stirrups when the beam was at the computed ultimate load assuming a rectangular distribution of stress in the concrete. The stirrups were tack welded together by means of 1/4"Ø rods. These rods extended over the end thirds and did not enter the middle third of the beams as shown by Fig. 2 and Fig. 7.

D. Beams of Varying Depth - Six beams having various effective depths were made to study the effect of the depth on the design strength.

Eight stirrups were used in each end of these beams. In every case the longitudinal steel consisted of four 3/8" Ø deformed bars placed one inch from the bottom of the beam. The various depths of the beams were obtained by building up the metal forms to the required depth with two-inch lumber and in one case reducing the depth with a screed.

E. Compression Steel - Three beams were made with compression reinforcing. The size of these test beams was made the same as the beams in the major portion of this program. Four 5/8"Ø deformed bars were used for the tension reinforcing as used in the three beams numbered 244 (with no compression reinforcing). Twelve stirrups were used in each end of the beam in the same manner as shown in Fig. 2. In addition, two stirrups were placed five inches apart in the center of the beam to keep the compression steel from buckling and were wired to the compression steel. Upon testing it was found that these stirrups had shifted to one side or the other during the placing of the concrete. The compression steel used for beams 244-C1, 244-C2, and 244-C4, was

1, 2, and 4, 3/8"Ø deformed bars respectively. The center of gravity of the compression steel was set 1-1/4 in. from the top face. The physical properties of the compression steel was the same as that of the tensile steel and stirrups. (See Table II).

F. Forms - After the stirrup assemblies had been slipped over the longitudinal steel assembly; the entire assembly was placed in oiled steel forms. The steel assemblies are shown in Fig. 2 and the steel forms in Fig. 9.

The forms were practically water-tight. The sides and ends were made of six-inch steel channel. One side and end were welded to the steel base while the other side and end were bolted to the base. Two hairpin shaped rods were embedded in the concrete at each end of the beams to facilitate the removal and handling.

G. Curing - In all cases the concrete was removed from the moulds at the age of one day and placed in the moist room for 27 days.

H. Beam Test Procedure - The beams were kept moist after their removal from the moist room until they were tested. Previous to testing the load-points and position of supports were located on each beam.

The testing rig had to be specially designed because of the limited space available in the testing machine. The testing apparatus is shown by Fig. 10 and Fig.11.

A solid steel block three inches deep, six inches wide and forty-seven inches long, resting on the testing machine served as a means to extend the length of the table. Steel blocks were then welded at each end of this base 45 inches on centers. On the top of these blocks, one-inch rollers were placed to reduce any end restraint between the support and the beam. A steel plate 1/2 in. deep and 1-1/2 in. wide was placed between the rollers and the beam in order to distribute the load. A layer of Celotex was placed between the beam and plate to protect the concrete.

A layer of Celotex was placed at the third-points and this was covered by a 1/2 by 1-1/2 in. plate and 1-in. rollers, on which rested a loading head. The loading head was made of two 40,000-lb. capacity railroad car springs placed symmetrically above the third-points. These springs enabled the observer to follow the load at all times and gave the same effect as a dead load. In this way the yield point of the steel within the beam could be found by the drop of the beam.

Since it was not possible to use a spherical bearing block with this loading device, a method was devised which produced an equal distribution of the load. To do this a rectangular shaft was dropped through the head of the testing machine through which a 1-1/4 inch tool steel pin was placed. As the head moved down it rested upon the steel pin permitting the loading device to rotate in a similar manner to a that of a spherical bearing block.

At each end of the beam, over the center of the supports and at the computed neutral axis;a deflection frame was supported by two 1/4-in. pins. One end of the deflection frame was notched so that it could slide freely over the pin. An 1/10,000 in. Ames dial was placed in the center of the beam on the deflection frame to read the total deflection of the beams.

The load was applied through the above described springs as the testing machine head moved down. The deflections were read "on the run" at thousand pound increments or less depending upon the ultimate strength of the beam.

The yielding of the steel and the progression of cracks were noted. Failures of the concrete occurred without warning in the case of the high strength concrete and high percentages of steel, and produced quite a resounding noise. Fig. 12 to 17 inclusive show typical beam failures.

IV - TEST DATA

A. Failure of the Beam Specimens - The load history of the beams can best be shown by load-deflection curves. A number of load-deflection curves are shown by Fig. 18 to 30 inclusive.

The first general break in these curves occurs at low loads and indicates the failure of the concrete in the tensile fibers. The second break in the curve near the ultimate load of the beams occurs at the time when the steel yields or the concrete crushes. The steel yielded first in the cases which did not have "balanced design" or over design in the steel; and the concrete crushed for those cases in which the steel was over-designed.

Both structural yielding of the beam and the ultimate load could be observed by a drop of the testing machine balance arm. Typical beam failures are shown in Fig. 12 to 17 inclusive. Concrete beam failures of specimens with cement-water ratios of 1.00, 1.25, 1.50, and 1.75, are represented by Fig. 12, 13, 14, and 15 respectively.

In the beams having a small percentage of steel, a considerable deflection would occur before the concrete failed. Failure, in the beams with high percentages of steel, occurred very suddenly.

The beams of the 200 series, i.e., those beams with concrete having a cement-water ratio of 1.25, were made in triplicate. The tests, in general, were in such close agreement with one another that it was decided for the remaining three cement-water ratio groups to make but one specimen for each steel area. The type of failure was not at all affected by the strength of concrete but the beams of higher strengths of concrete broke with more noise than the lower concrete strength beams.

The plain concrete beams did not give the expected results because the beams did not all fail between the load points where the bending stresses were maximum.

The results of the three beams with compression reinforcing are given in Table V. As shown by Fig. 29 the load-history of these beams was not very different from the beams without compression reinforcing, except that the ultimate load was increased by the addition of the compression steel. When the tensile steel was over-designed the compression steel buckled causing the surrounding concrete to spall off. The failures obtained in beams with compression steel are shown in Fig. 16.

The beams with varied depths provided no unusual test observations. Fig. 30 indicates quite clearly the load-history of the beams and in Table V can be found a summary of the data obtained on these beams of various depths.

B. Materials - The data obtained for the physical properties of the steel are given in Table II and for the various cement-water ratio concretes in Table I. A standard testing procedure was followed in the testing of the concrete and steel. The resulting modulus curve for concrete having a cement-water ratio of 1.25 is shown in Fig. 31. This curve is the average of three tests.

V - DISCUSSION OF TEST RESULTS

A. General - The test results were, in general, as uniform as might be expected. This is particularly true of all the beam tests. In those cases in which the beams were made in triplicate the results did not deviate very much from the average value.

The results of the control cylinders and plain concrete beams had more variation than might be expected. The plain concrete beams are by their very nature less dependable as test specimens than are reinforced concrete beams, since their strength depends upon the tensile strength of the concrete.

B. Physical Tests on the Concrete - Because vibration was used in the manufacture of the beams it was desirable to find the effect of vibration on the strength of concrete. Cylinders were made as has previously been described.

The results of the individual tests indicated that the 6 by 12-in. cylinder vibrated by the standard established during the course of these tests gave lower but more consistent results than did either of the other groups of cylinders.

The relationship between the compression strength and the cement-water ratio indicates that the results were close to the customary relationship expected from this type of test (Fig.32). By examining this curve and Tables I and

VI, it can be seen that in every case the 3 by 6-in. rodded cylinders had the highest compressive strength, the 6 by 12 in. rodded cylinders were nearest to the average strength of all cylinders and the 6 by 12-in. vibrated cylinders, although the lowest, did not vary as much from their own average value as the 3 by 6-in. rodded cylinders.

To account for this difference one should go back to the method of making the cylinders. The author believes that the water leakage per unit of volume was greater in the case of the 3 by 6-in. cylinders than in either of the other two cylinder sizes. This leaking also accounts for part of the difference between the two 6 by 12-in. groups. The moulds were bolted to metal base plates in the case of the vibrated group, whereas, the mould merely rested on the base plates in the rodded cylinder groups. As a result of water leakage the cement-water ratio would become greater and produce a stronger concrete.

The modulus curve of the concrete, Fig. 31, is included in case any future studies wish to be made using the modulus of the concrete. The data from which the curve is drawn appears to be very consistent. The method used in obtaining this curve has the advantage of being simple to perform yet with speed and consistency.

C. Beam Tests and "Balanced Design" Data - The data obtained from the beam tests show very clearly, in the case of the concrete with cement-water ratios of 1.00 and 1.25, just what the percentage of steel results in a "balanced design".

The percentage of steel for "balanced design" can most easily be obtained by studying the curves in which the ultimate load is plotted against the percentage of steel. The curves in Fig. 33 to 41 inclusive show a break at a certain percentage of steel. This point or break occurs at the time when the steel yield-point strength and compression strength of the concrete are reached at the same time. Below this point the steel yields before the concrete fails; while above the break the concrete crushes before the steel reached its yield point. Hence, the breaking point is the measure of the per cent of steel for "balanced design".

Examining the percentages of steel to produce "balanced design" in the figures referred to above and substituting these percentages in the formula

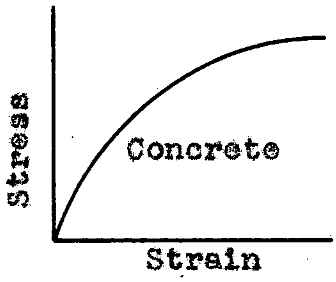
$$p_m = C'\frac{f_c}{f_s}$$

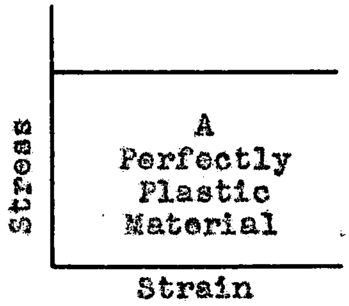
with the correct values of "f_c" and "f_s" we will find that C' is approximately 0.47.

In the case of the high cement-water ratios, or higher strength concretes, the maximum percentage of steel for "balanced design" is not such a distinct point as we have in the lower cement-water ratio concretes. This is shown by a rounding off of the curves in Fig. 42 and 43. If an estimate is made based on the data obtained for the maximum percentage of steel with concrete having a cement-water ratio of 1.00 or 1.25, it can be seen that there was not enough steel placed in the beams of cement-water ratios 1.50 and 1.75 to reach a "balanced design". Perhaps in many cases such a high percentage of steel would be impractical because of the difficulty which might be encountered in arranging the steel and placing the concrete.

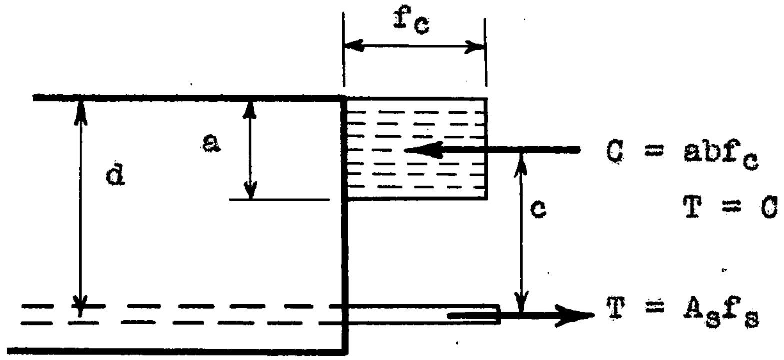
Fig. 44 compares the maximum steel ratios allowed by the straight line method, the parabolic method, Whitney's method, and an equation derived from the observed data. Whitney's method, an empirical method, has been based on actual tests performed by Slater and Lyse of Lehigh University[2], by Humphrey and Losse of the Bureau of Standards[1], and by the U.S.Geological Survey[1]. Perhaps this is why the observed results are in line with Whitney's conclusions.

The proposed method partially derived from material in this thesis is given below. The first assumption made is that of rectangular distribution of stress, i.e., there is a rectangular block which has an equal distribution of stress over a cross section of the concrete near the top of the beam.





This assumption is made because concrete behaves to some extent as a plastic material; and the assumption of the stress-strain diagram of a perfectly plastic material gives results which are in close agreement with the breaking strength of beams. (See Nadil's Plasticity). Upon this assumption and with the percentage of steel for "balanced design" obtained above, the proposed method is based.



$$M_s = A_s cf = M_c = abf_c c$$

$$A_s f_s = abf_c$$

$$\frac{a}{2} = \frac{A_s f_s}{2bf_c}$$

$$M = \left(d - \frac{A_s f_s}{2bf_c}\right)A_s f_s = \left(d - \frac{A_s f_s}{2bf_c}\right)abf_c = \left(d - \frac{pf_s d}{2f_c}\right)pbdf_s$$

$$\frac{M}{bd^2} = pf_s(1 - \frac{pf_s}{2f_c}) \qquad (1)$$

Fig. 45, 46, 47, and 48 were computed using the above mentioned methods. Again it is seen that Whitney's method, the proposed method and the observed data agree very closely. Working stresses of forty per cent of the yield-point strength of the steel and forty per cent of the compressive strength of the concrete were used in each method. The observed data were plotted with no safety factor and also with forty per cent of the observed $\frac{M}{bd^2}$ values.

The straight-line method of design needs no explanation, since it is the prevailing method. Whitney's method assumes a rectangular stress distribution in the compression face of a reinforced concrete beam. He sets the design strength of the concrete to be 0.85 of the cylinder strength. Hence there is a striking similarity between Whitney's method and the proposed one. The "balanced design" percentage of steel given by Whitney is

$$p_{max} = 0.456 \frac{f_c}{f_s} \qquad (2)$$

A factor of safety can be taken for both steel and concrete and the design made on that basis. The advantage of his method lies in the simplicity of application and derivation. Modulus ratios, "k" and "j" values are abandoned. In addition to simplicity it gives results closer to the observed data than the straight line or parabolic methods.

The proposed method obtained from the observed data parallels Whitney's method but is simpler and yet retains all of the advantages of the latter. In the proposed method the coefficient 0.85 of the cylinder strength is omitted since it had little effect upon the design. In designing by this method with a rectangular stress distribution, the cylinder strength divided by the factor of safety gives us the value of f_c to use in design. The observed data limited the maximum steel ratio for "balanced design" at $p_{max} = 0.47 \frac{f_c}{f_s}$. This value proved to be the best for concrete strengths 3000 p.s.i. or lower but was excelled by Whitney's $p_{max} = 0.456 \frac{f_c}{f_s}$ in the higher strength concretes. The author believes that the best value is somewhere between $0.456 \frac{f_c}{f_s}$ and $0.47 \frac{f_c}{f_s}$, and proposes that the equation for the maximum steel ratio as

$$p_{max} = 0.46 \frac{f_c}{f_s} \qquad (3)$$

The conventional straight-line method will not be given in detail in this paper. (See Turneaure and Maurer, reference number 3 for derivation). The derivation of the formula for the percentage of steel for a "balanced design" using the straight line distribution of stress results in the formula:

$$p_{max} = \frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)} \tag{4}$$

In a similar fashion using the parabolic distribution of stress one would obtain the equation:

$$p_{max} = \frac{2/3}{\frac{f_s}{f_c}\left(\frac{f_s}{2nf_c}+1\right)} \tag{5}$$

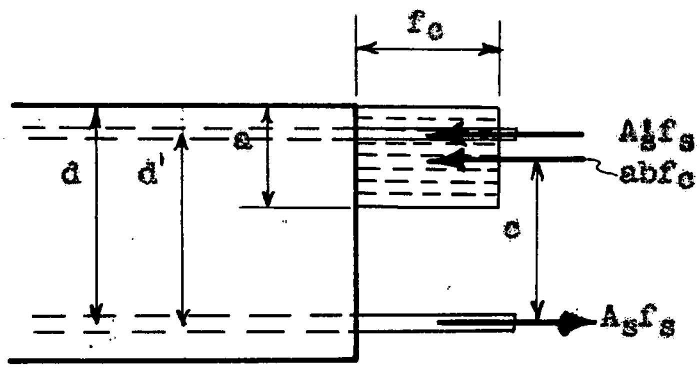
Using the observed data as a basis for comparison, beams designed with the straight line method contain only forty per cent as much steel as beams designed by the proposed method. This assumes the strength of concrete and factor of safety are equal in both cases.

From the standpoint of factor of safety at balanced design, the present A.C.I. method gives the concrete a factor of safety 2.5 times as great as the factor of safety in the steel. The steel has in the past been more dependable as a material than concrete, but the time has come when the science of concrete making has enabled control, with adequate supervision, to keep pace with design. Concrete gains in strength with the passing of time under normal conditions, but steel retains it's original properties. Plastic flow relieves stresses in the concrete and increases steel stress. These present other arguments for equalizing the safety factors of these materials. One should remember that higher stresses in the concrete would allow still more plastic flow to take place. This would be an argument against changing the true factor of safety of the concrete.

By setting a definite safety factor in the proposed method ~~or~~ in Whitney's method there is an "overall" factor for the beam as a structural unit. The straight-line method results in a high factor of safety for the concrete and the desired factor in the steel.

The tests made on beams with compression reinforcing and the resulting computations show that the proposed theory is also adaptable to beams reinforced with compression steel. Fig. 49 illustrates how closely the additional strength due to the compression steel can be approximated. The method requires the determination of the strength of a beam without compression steel and then adding the strength contributed by the compression steel. The tensile steel area must equal the area required for balanced design with no compression steel plus an area equal to the area of compression steel used.

Compression Steel (Proposed Method)



The following expression has been derived previously (see Equation 1).

$$\frac{M}{bd^2} = pf_s(1 - \frac{pf_s}{2f_c})$$

For "balanced design", $p = 0.46 \frac{f_c}{f_s} \therefore (1 - \frac{pf_s}{2f_c}) = 0.77$

$$M = pf_s bd^2(0.77) + A_s' f_s d'$$

The total steel in the compression side will be A_s'. The total steel area in the tensile side will be the percentage required for "balanced design" plus an amount of steel equal to the compression steel, or:

Total area of tensile steel = $pbd + A_s'$

That the proposed method is satisfactory for varying depths of beam is shown by Fig. 50 and Fig. 51 which compare the computed and observed data for various beam depths. The observed data show that the ultimate loads lie slightly above the computed curve for the deepest beams and very close to the computed curve for shallow beams. This can be explained by the fact that the reserve strength between the structural yield of a beam and the ultimate load of the beam are closer together for high percentages of steel than for low percentages of steel. One curve was drawn using the structural yield load in place of the ultimate load and this resulted in a better correlation with the computed curve.

D. <u>Comparison with Other Tests</u> - In a recent investigation carried on at the Fritz Engineering Laboratory by the author[20], 32 rectangular reinforced concrete beams were tested. These beams were reinforced with high yield-point steel of several different types. The beams had an effective depth of twelve inches, a width of twelve inches and the distance center-to-center of supports was equal to nine feet. Fig. 52 shows how closely the proposed method of design compares with the structural yield load of these 32 beams. Fig. 52 illustrates the same points as Fig. 51 except that the computed load is plotted against the ultimate load. The reader will notice a reserve strength in the ultimate loads over the computed values in Fig. 53 which is not the case when comparing the structural yield load with the computed values. This can be explained by a point mentioned previously, i.e., for low percentages of steel the structural yield of the beam is lower than the ultimate strength. The difference between ultimate load and structural yield load (yield point x area of steel) diminishes as the per cent of steel approaches the balanced design percentage. In short, the proposed method of design would apply to the 32 rectangular beams with high yield-point steel using any desired factor of safety.

The proposed method has been compared with other test data dating back as far as 1904 and up to the present time. The computed results using the proposed method compare very well with observed data in all cases. The fluctuations between observed and computed data depend to a very large extent upon the assumptions or estimates which must be necessarily made on the properties of the materials.

The average ratio of observed to computed ultimate strength values for 34 beams taken at random from various publications was 1.011. Whether this would be lowered or raised by more complete knowledge of the previous tests is debatable, but it is quite certain that in every consideration the rectangular method is just as adequate and far more simple to apply than the straight-line method.

E. Deflection Observations - The deflection readings were taken "on the run" and gave very consistent results as shown in Fig. 18 to 30. The first break in the load-deflection curves near the low loads indicates failure of the concrete in tension. The curve advances approximately in a straight line to the next break, marking the initial yielding of the steel which eventually causes failure.

In beams with low percentages of steel the deflection at any load is far greater than in the high percentages as shown in Fig. 18 to 30 inclusive. It is evident from an examination of these figures that when the deflection at structural yielding of the beams and deflection at the ultimate reach the same value the critical point for a "balanced design" has been reached. It should be noted that the average deflection at "balanced design" percentages for all strengths of concrete was approximately 0.200 to 0.210 of an inch or 0.04 to 0.05 per cent of the span length for the beams with no compression reinforcing and five inches effective depth.

F. Cracking of the Beams - The beams are too small to make any exact study of the progression of cracks; however, the first signs of cracking were noted. The load at which first visible cracking occurs has been plotted against the yield strength of the steel in the beams. A curve was also drawn representing the ultimate load. By comparing these two average lines drawn in Fig. 54 to 57 inclusive it will be noticed that the visible cracking occurred at varying percentages of the ultimate load but was always over fifty per cent of the ultimate load. This point may be of considerable consequence in designing certain types of beams.

G. Economic Study - The variety of areas of steel, concrete strengths and beam depths afforded the opportunity to study the question of relative cost of various sections. A steel cost of $0.01 per cubic inch of concrete was assumed; this figure is approximately equal to $0.035 per pound (placed). The cost of forms, placing, curing, finishing, etc., have not been included in the cost of the concrete. These items should be constant except for the cases of beams of various depths. For cement-water ratios of 1.00, 1.25, 1.50, and 1.75 the concrete was computed to cost \$3.01, \$3.27, \$3.62, and \$3.92 per cubic yard respectively. The cost of stirrups has not been included since for any particular ultimate load the amount of stirrup material would be a constant. In order to make the figures of some magnitude, a section was used having the cross section equal to the beams in this report multiplied by one thousand.

Fig. 58 illustrates the result of the investigation of the question on the most economical sections. In this curve the cost per pound of ultimate load carried is compared to the ultimate load. The diagram shows that concrete in the neighborhood of 3000 p.s.i. is quite suitable for the most economical design. It is evident that in this range of strengths the "balanced design" is the most economical. This was not true for concrete strengths over 3000 p.s.i.

Form costs must be considered when one varies the depth of the section. If form costs are relatively low the most economical way to increase the strength of a beam is by increasing the depth.

The reader will note that to use compression steel is more economical than to use concrete of very high strength. There are concrete strengths, however, which are more economical than the use of compression steel.

VI - SUMMARY AND CONCLUSIONS

The following conclusions are based upon the test results presented in this thesis:

1. For "balanced design" the maximum steel ratio varies according to the formula $p_{max} = 0.46 \frac{f_c}{f_s}$.

2. Whitney's method ($f_c' = 0.85 f_c$ with rectangular distribution) more closely approximates actual test results than the parabolic or straight-line distribution methods. Whitney's method and the proposed method give equally good results. The proposed method falling closer to the observed data in the concrete strengths around 3000 p.s.i.

3. The proposed method is in closer agreement with actual test data than the common straight-line distribution method.

4. The proposed method has the advantage of being the most simple to apply and the easiest to understand of all the design methods studied.

5. The results of 102 beams tested in this thesis, 32 large beams in a program carried on by the author and 34 beams selected at random from test data of early tests at other laboratories indicates that the proposed method is adaptable to every size and depth of beam.

6. At "balanced design" the straight-line theory with it's present limitations uses only forty per cent as much steel as could be used with the present factors of safety.

7. The yield point of compression steel is reached before buckling takes place in the compression steel.

8. The proposed method includes the use of compression steel, affording simplicity of design.

9. The reserve strength between the structural yield load of a beam and the ultimate strength is greater at low percentages of steel than it is at the high percentages. Diminishing as the percentage of steel approaches a "balanced design".

10. The test results show that additional steel over and above "balanced design" does not add material strength to a beam.

11. Additional steel over that required for a "balanced design" will reduce the deflection for any given load and may in many cases be an important consideration in design.

12. The first visible cracks usually occurred at loads well over fifty per cent of the ultimate load.

13. Vibrated test cylinders in this report gave more consistent results than did the rodded specimens.

14. The amount of leakage for control cylinders should be controlled or eliminated for comparable and consistent results.

NOMENCLATURE

p_m = steel ratio for "balanced design"

C' = a constant

f_c = cylinder strength of concrete, pounds per square inch

f_s = yield-point strength of steel, pounds per square inch

d = effective depth of beam, inches

a = depth of concrete assumed to contribute full compression strength

b = breadth of rectangular beam

e = lever arm of steel reinforcement

C = total compression in concrete

T = total tension in steel

M = bending moment or resisting moment, in general

M_c = resisting moment of concrete

M_s = resisting moment of steel

A_s = area of tensile steel

A_s' = area of compression steel

d' = lever arm between tensile and compression steel

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VITA

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Hale Sutherland

Head of the Department of Civil Engineering

Inge Lyse

Formerly Research Professor of Engineering Materials

Bruce Johnston

Assistant Professor of Civil Engineering

Sylvanus Becker

Associate Professor of Civil Engineering

Albert Haring

Associate Professor of Economics

TABLE I
CONCRETE MATERIALS USED FOR THE VARIOUS CEMENT-WATER RATIOS

c/w Ratio	Cement lb.	Water* lb.	Sand lb.	Stone lb.	Average Compressive Strength All Sizes p.s.i.	Number of Cylinders in Average
1.00	27.1	29.8	108	163	1970	9
1.25	33.8	29.8	106	160	3060	33°
1.50	40.6	29.7	104	156	4750	9
1.75	47.4	29.7	102	153	5870	9

DATA ON COMPRESSION STRENGTH OF CYLINDERS

c/w Ratio	Number and Size of Cylinders (Vibrated)	28-day Compressive Strength p.s.i.	Number and Size of Cylinders (Rodded)	28-day Compressive Strength p.s.i.	Number and Size of Cylinders (Rodded)	28-day Compressive Strength p.s.i.
1.00	3, 6" x 12"	1810	3, 6" x 12"	1920	3, 3" x 6"	2170
1.25	3, 6" x 12"	3220	3, 6" x 12"	3340	3, 3" x 6"	3360
1.50	3, 6" x 12"	4650	3, 6" x 12"	4650	3, 3" x 6"	4950
1.75	3, 6" x 12"	5470	3, 6" x 12"	5690	3, 3" x 6"	6360

* One per cent of weight of aggregate added for absorption.

° Twenty-four additional vibrated cylinders included.

NOTE: Design based on 325 lb. of water per cu yd. for workability required.

TABLE II - PHYSICAL PROPERTIES OF THE STEEL

Bar Size	Yield Load lb.	Yield Stress p.s.i.	Ultimate Stress p.s.i.	Per Cent Elongation 2"	Per Cent Elongation 8"	Per Cent Reduction in Area	Number of Specimens Tested
1/4"Ø def.	2,670	55,200	76,500	26.5	18.1	63.9	5
3/8"Ø def.	5,880	53,400	82,500	26.5	19.1	50.9	10
1/2"Ø def.	9,290	48,100	78,000	33.5	22.8	57.0	5
5/8"Ø def.	14,690	48,100	78,600	31.5	22.3	45.5	5
3/4"Ø def.	21,970	50,600	77,700	39,0	25.6	48.4	3

TABLE III - SUMMARY OF DATA ON BEAMS HAVING A CEMENT-WATER RATIO OF 1.25

Beam No.	Size and Kind of Bars	Yield Strength of Steel lb.	Average Structural Limit Load lb.	Average Ultimate Load lb.	Deflection at Structural Load lb.	Deflection at Ultimate Load lb.	Per Cent Steel	Area of Steel sq. in.	Stirrups in Each End of Beam
221	2, 3/8"Ø def.	11,760	6,480	7,490	0.135	0.862	0.88	0.22	6
222	4, 3/8"Ø def.	23,520	12,690	12,800	.179	.420	1.76	.44	6
223	6, 3/8"Ø def.	35,280	17,920	18,000	.229	.363	2.64	.66	6
224	8, 3/8"Ø def.	47,040	17,640	17,640	.233	.233	3.52	.88	8
225	10, 3/8"Ø def.	58,800	18,160	18,160	.225	.225	4.40	1.10	9
241	1, 5/8"Ø def.	14,690	8,700	8,810	.144	.545	1.20	.30	6
242	2, 5/8"Ø def.	29,380	16,090	16,100	.191	.306	2.44	.61	6
243	3, 5/8"Ø def.	44,070	19,020	19,020	.226	.226	3.68	.92	8
244	4, 5/8"Ø def.	58,760	19,300	19,300	.198	.198	4.88	1.22	9
211	4, 1/4"Ø def.	10,680	6,980	7,150	.131	.373	0.76	.19	6
212	8, 1/4"Ø def.	21,360	12,970	13,180	.176	.388	1.56	.39	6
213	12, 1/4"Ø def.	32,040	17,900	17,910	.223	.265	2.32	.58	6
214	16, 1/4"Ø def.	42,720	19,240	19,240	.238	.238	3.08	.77	8
215	20, 1/4"Ø def.	53,400	19,150	19,150	.255	.255	3.88	.97	8
231	1, 1/2"Ø def.	9,290	5,540	6,480	.120	1.250	0.76	.19	6
232	2, 1/2"Ø def.	18,580	11,030	11,340	.166	.450	1.56	.39	6
233	3, 1/2"Ø def.	27,870	15,410	15,730	.183	.331	2.32	.58	6
234	4, 1/2"Ø def.	37,160	17,910	17,910	.320	.320	3.08	.77	8
235	5, 1/2"Ø def.	46,450	19,580	19,580	.243	.243	3.88	.97	8
251	1, 3/4"Ø def.	21,970	11,900	12,300	.181	.357	1.72	.43	6
252	2, 3/4"Ø def.	43,940	19,490	19,490	.237	.237	3.48	.87	8
253	3, 3/4"Ø def.	65,910	19,980	19,980	.202	.202	5.20	1.30	9
200	0 bars	0	1,830	1,830	.010	.010	0	0	0

c/w = 1.25

All above figures are the average of three specimens

TABLE IV - SUMMARY OF DATA ON BEAMS HAVING CEMENT-WATER RATIOS OF 1.00 and 1.50

Beam No.	Size and Kind of Bars	Yield Strength of Steel lb.	Average Structural Limit Load lb.	Average Ultimate Load lb.	Deflection at Structural Load lb.	Deflection at Ultimate Load lb.	Per Cent Steel	Area of Steel sq in.	Stirrups in Each End of Beam
100	0 bars	0	1,310	1,310	0	0.005	0	0	0
121	2,3/8"Ø def.	11,760	6,450	6,650	.158	.295	0.88	0.22	6
122	4,3/8"Ø def.	23,520	11,230	11,380	.233	.312	1.76	.44	6
123	6,3/8"Ø def.	35,280	11,910	11,910	.218	.302	2.64	.66	6
124	8,3/8"Ø def.	47,040	12,800	12,800	.201	.235	3.52	.88	6
125	10,3/8"Ø def.	58,800	13,250	13,250	.200	.260	4.40	1.10	6
141	1,5/8"Ø def.	14,690	7,840	8,070	.150	.270	1.20	.30	6
142	2,5/8"Ø def.	29,380	12,440	12,440	.242	.320	2.44	.61	6
143	3,5/8"Ø def.	44,070	11,980	11,980	.195	.295	3.68	.92	6
144	4,5/8"Ø def.	58,760	13,800	13,800	.184	.244	4.88	1.22	6
				c/w = 1.00					
300	0 bars	0	1,200	1,200	0	0.004	0	0	0
321	2,3/8"Ø def.	11,760	7,350	8,520	.118	1,250	0.88	0.22	6
322	4,3/8"Ø def.	23,520	13,760	14,170	.157	.468	1.76	.44	6
323	6,3/8"Ø def.	35,280	19,300	19,600	.200	.395	2.64	.66	6
324	8,3/8"Ø def.	47,040	22,000	22,000	.258	.258	3.52	.88	8
325	10,3/8"Ø def.	58,800	23,920	23,920	.230	.230	4.40	1.10	9
341	1,5/8"Ø def.	14,690	9,320	10,210	.148	.710	1.20	.30	6
342	2,5/8"Ø def.	29,380	16,030	16,610	.172	.322	2.44	.61	6
343	3,5/8"Ø def.	44,070	22,200	22,200	.210	.210	3.68	.92	8
344	4,5/8"Ø def.	58,760	25,580	25,580	.224	.224	4.88	1.22	9
				c/w = 1.50					

All the Above Figures are of a Single Test Specimen

TABLE V - SUMMARY OF DATA ON BEAMS HAVING CEMENT-WATER RATIOS OF 1.25 AND 1.75
(Compression Steel and Various Depth Beams)

Beam No.	Size and Kind of Bars	Yield Strength of Steel lb.	Average Structural Limit Load lb.	Average Ultimate Load lb.	Deflection at Structural Load lb.	Deflection at Ultimate Load lb.	Per Cent Steel	Area of Steel sq in.	Stirrups in Each End of Beam
400	0 bars	0	2,550	2,550	0.009	0.009	0	0	0
421	2,3/8"Ø def.	11,760	7,280	8,950	.120	1.250	0.88	.22	6
422	4,3/8"Ø def.	23,520	13,480	14,020	.149	.418	1.76	.44	6
423	6,3/8"Ø def.	35,280	19,980	20,540	.186	.342	2.64	.66	6
424	8,3/8"Ø def.	47,040	23,980	24,630	.231	.292	3.52	.88	8
425	10,3/8"Ø def.	58,800	28,270	28,270	.265	.265	4.40	1.10	9
441	1,5/8"Ø def.	14,690	8,900	10,550	.118	1.050	1.20	.30	6
442	2,5/8"Ø def.	29,380	16,570	17,570	.186	.438	2.44	.61	6
443	3,5/8"Ø def.	44,070	24,080	24,690	.203	.350	3.68	.92	8
444	4,5/8"Ø def.	58,760	28,960	28,960	.195	.245	4.88	1.22	9
				c/w = 1.75					
			Varying Depth Beams						
222x5	4,3/8"Ø def.	23,520	9,820	10,120	.250	.380	2.20	.44	8
222x7	4,3/8"Ø def.	23,520	16,770	16,770	.220	.480	1.47	.44	8
222x8	4,3/8"Ø def.	23,520	20,100	21,130	.130	.300	1.26	.44	8
222x9	4,3/8"Ø def.	23,520	22,800	24,280	.164	.594	1.10	.44	8
222x10	4,3/8"Ø def.	23,520	26,500	28,000	.106	.600	0.98	.44	8
222x11	4,3/8"Ø def.	23,520	29,130	31,000	.105	.600	0.88	.44	8
			Beams with	Compression Reinforcing					
244-C1	4,5/8"Ødef.	58,760	22,670	22,670	.238	.332	4.88*	1.22*	13
244-C2	4,5/8"Ødef.	58,760	25,260	25,260	.254	.348	4.88*	1.22*	13
244-C4	4,5/8"Ødef.	58,760	29,320	29,320	.252	.360	4.88*	1.22*	13
				c/w = 1.25					

*Does not include compression steel

TABLE VI - STRENGTH COMPARISON OF THREE TYPES OF COMPRESSION CYLINDERS

c/w Ratio	6" x 12" Vibrated p.s.i.	Compressive Strength		Mean p.s.i.	Per Cent Variation From Mean		
		6" x 12" Rodded p.s.i.	3" x 6" Rodded p.s.i.		6" x 12" Vibrated p.s.i.	6" x 12" Rodded p.s.i.	3" x 6" Rodded p.s.i.
1.00	1810	1920	2170	1970	- 8.1	- 2.5	+10.1
1.25	3220	3340	3360	3310	- 2.7	+ 1.0	+ 1.5
1.50	4650	4650	4950	4750	- 2.2	- 2.2	+ 4.1
1.75	5470	5690	6360	5870	- 6.8	- 3.1	+ 8.2
				Av.	- 4.95	- 1.95	+ 5.98

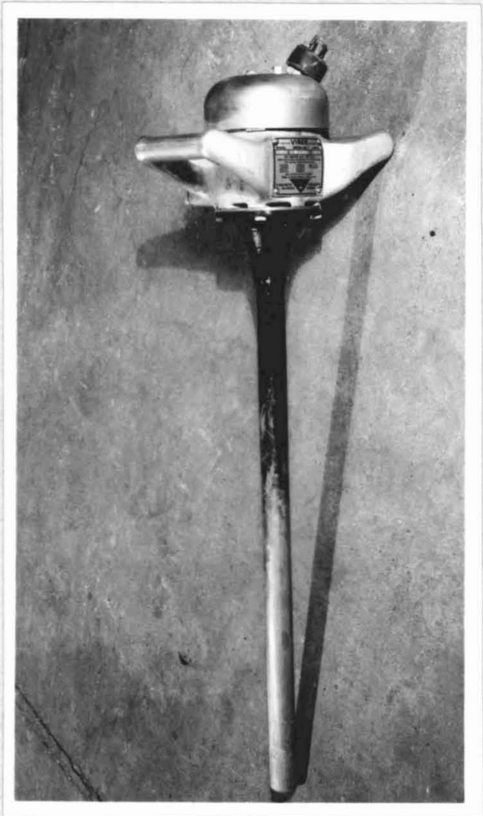


Fig. 1 - Vibrator Used In The
Manufacture Of The Specimens

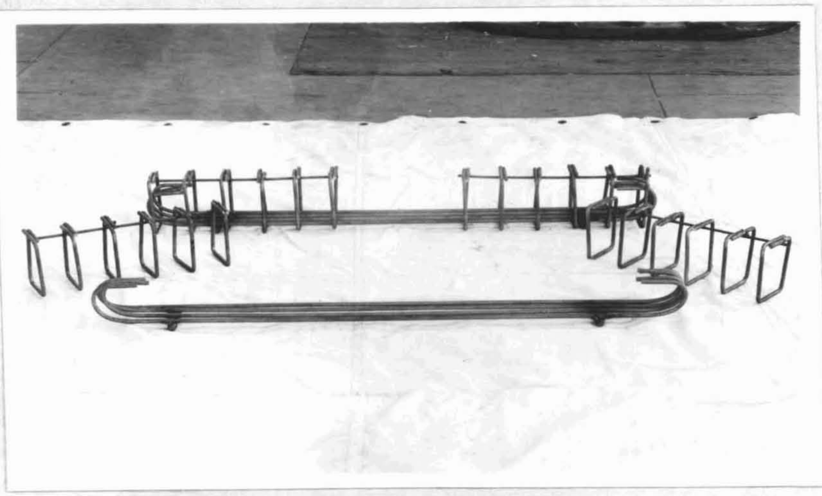


Fig. 2 - Assembly of Stirrups and Longitudinal Steel

Fig. 3 - Longitudinal Steel; Five Different Bar Sizes

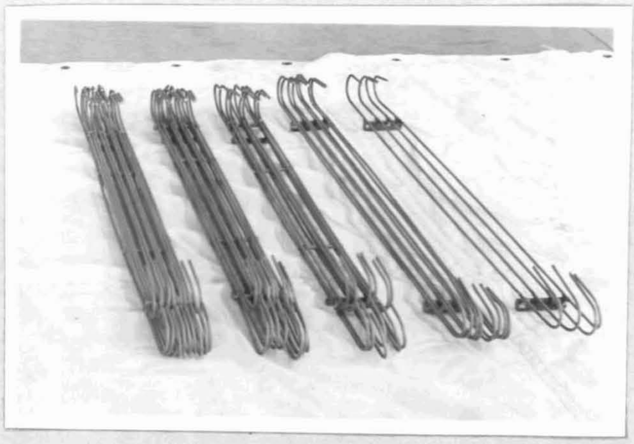


Fig. 4 - Assembly of 1/4"Ø Deformed Bars

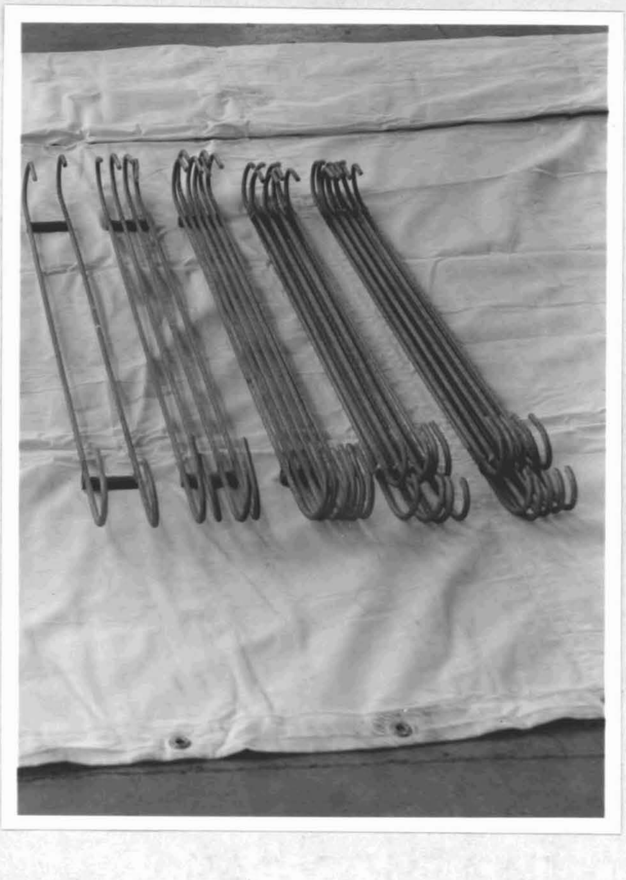


Fig. 5 - Assembly of 3/8"Ø Deformed Bars

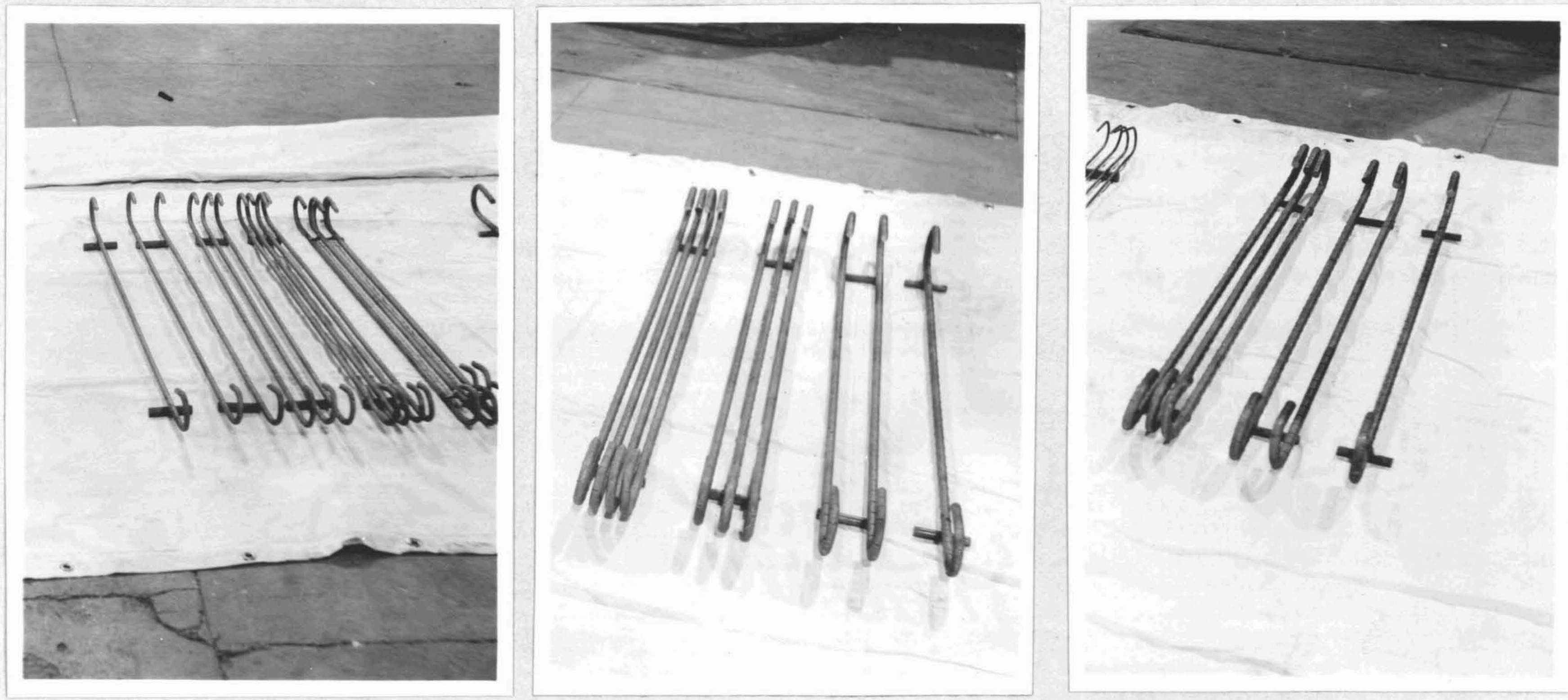
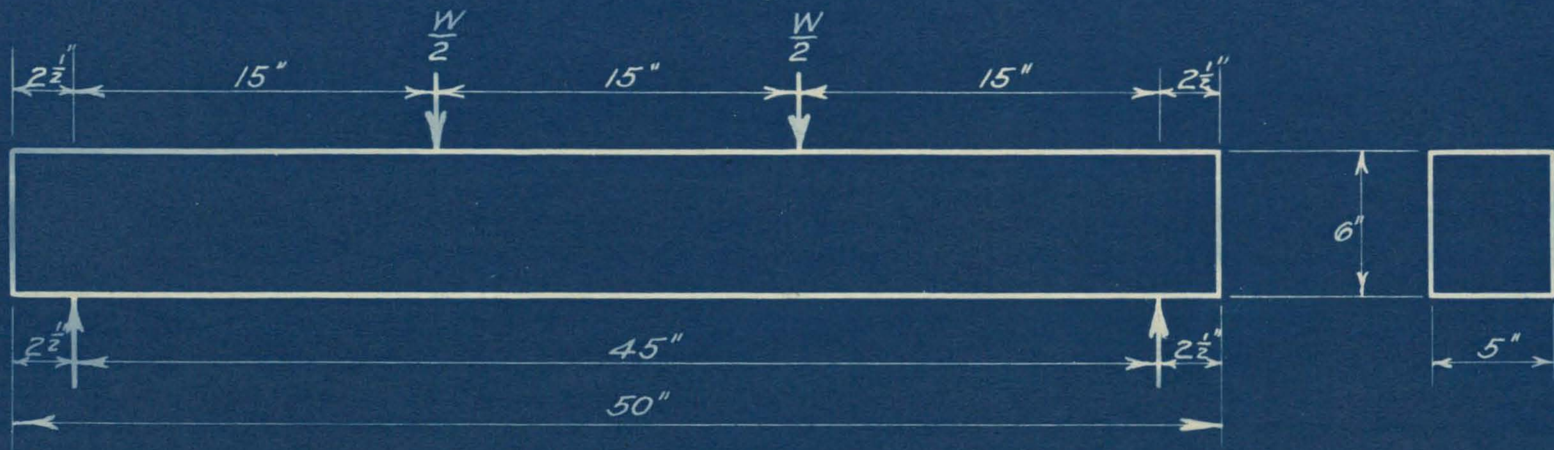


Fig. 6 - Assemblies of 1/2"Ø, 5/8"Ø, and 3/4"Ø Deformed Bars

Load Diagram



Steel Diagram

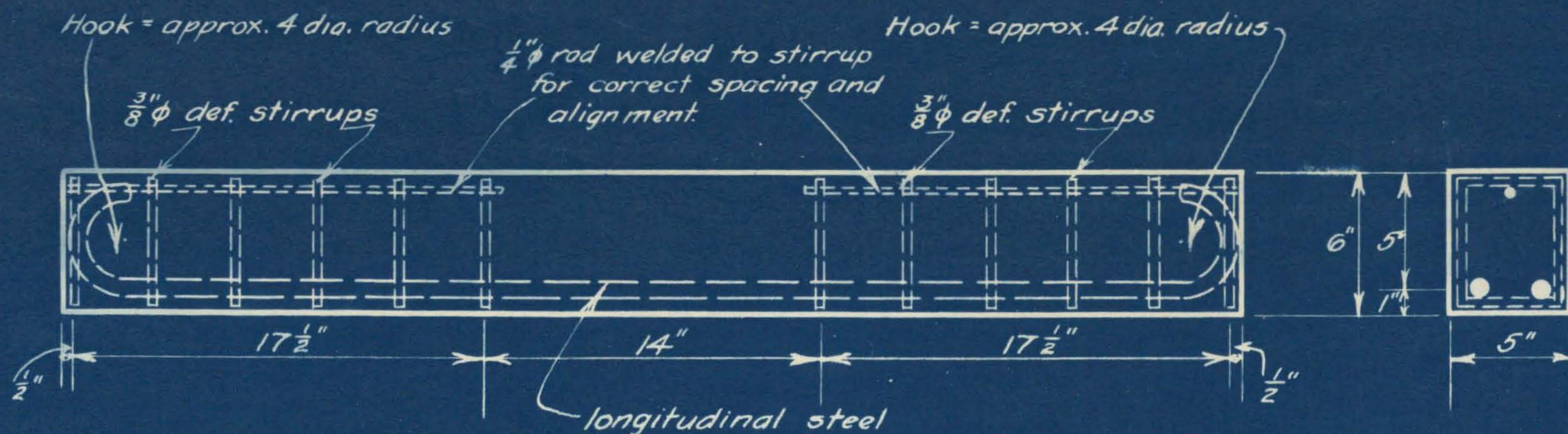


FIG. 7 — LOAD AND STEEL DIAGRAMS FOR THE BEAMS

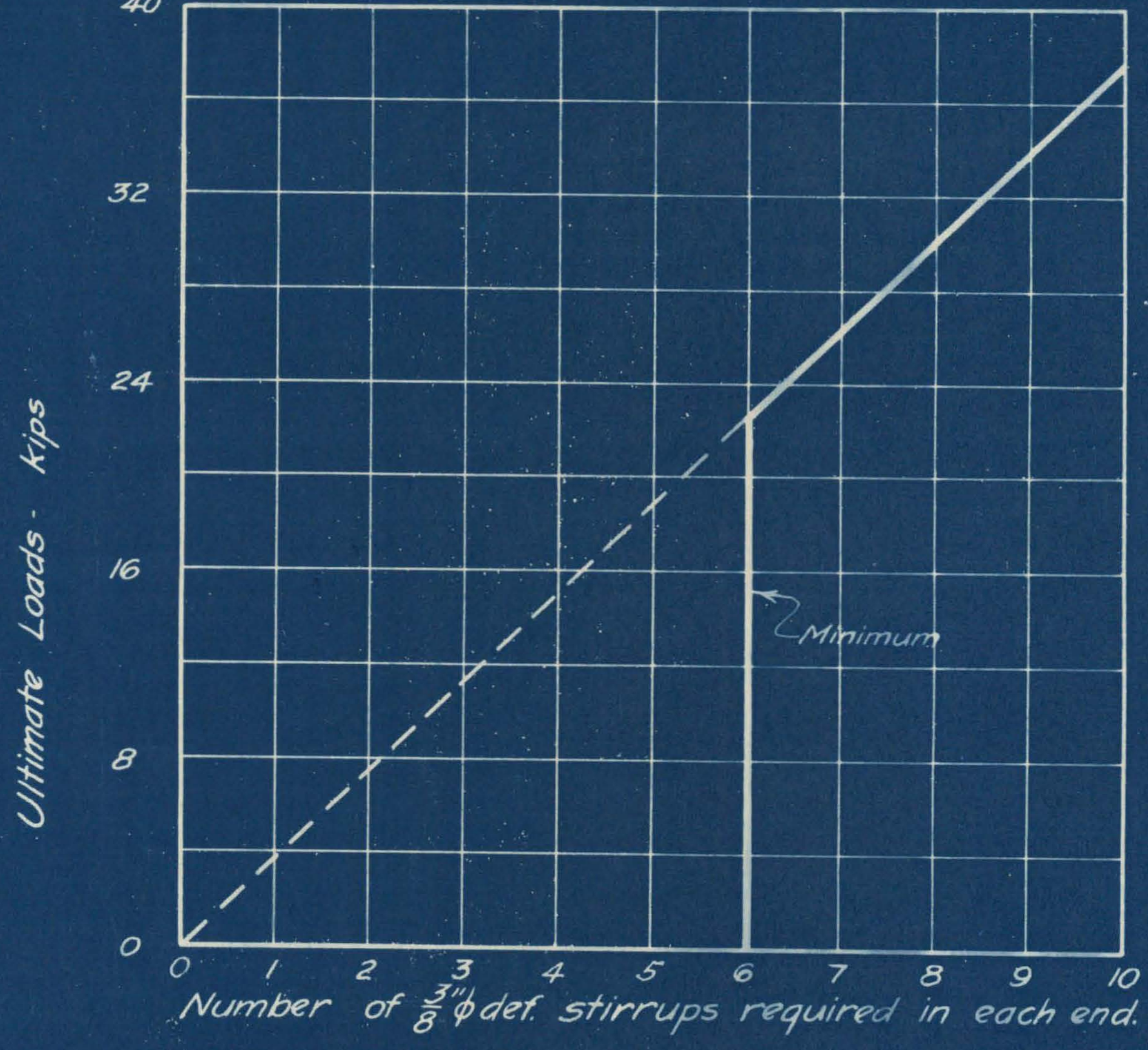


FIG. 8 - STIRRUP REQUIREMENTS FOR 6"x5"x50" Beams

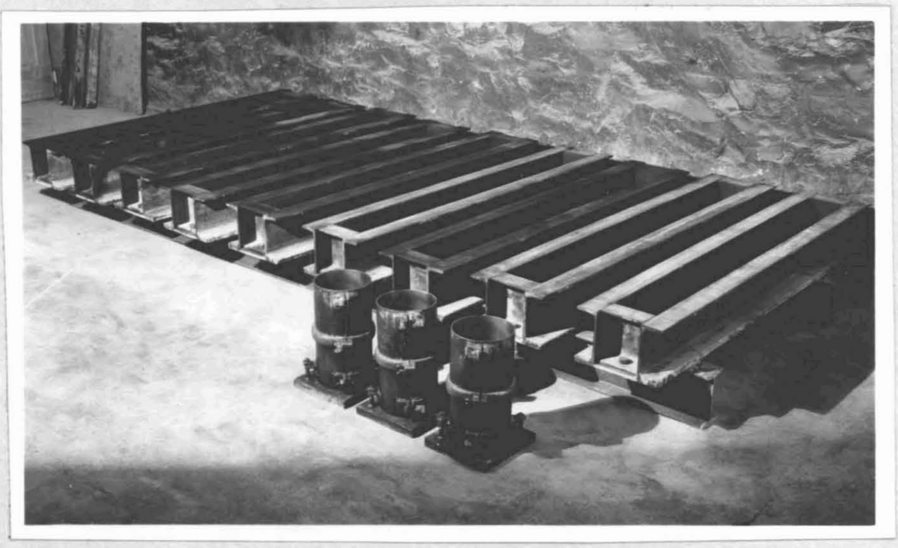


Fig. 9 - Steel Forms Used in Making
the Beams and Cylinders

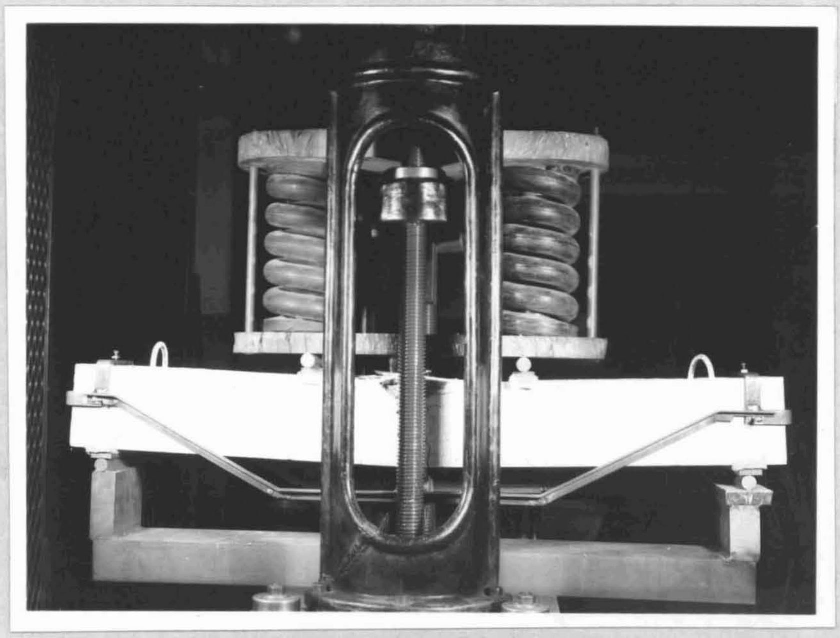


Fig. 10 - Transverse View of Beam
in Testing Machine

Fig. 11 - End View of Beam
in Testing Machine

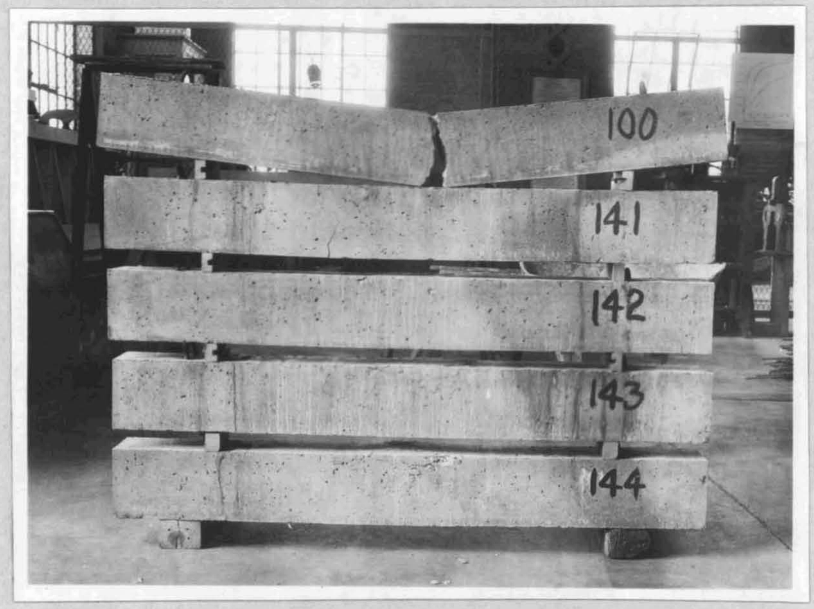


Fig. 12 - Beams Having a c/w Ratio of 1.00,
After Testing

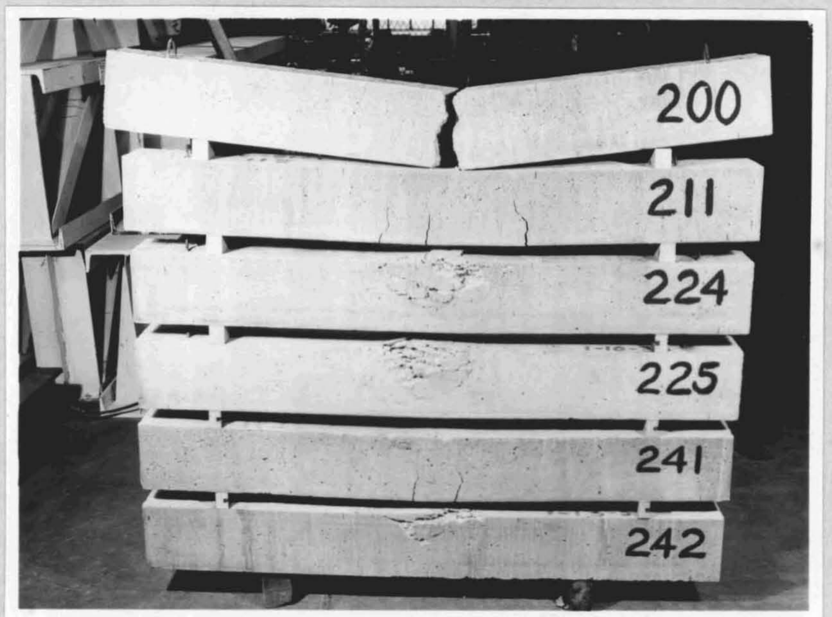


Fig. 13 - Beams Having a c/w Ratio of 1.25,
After Testing

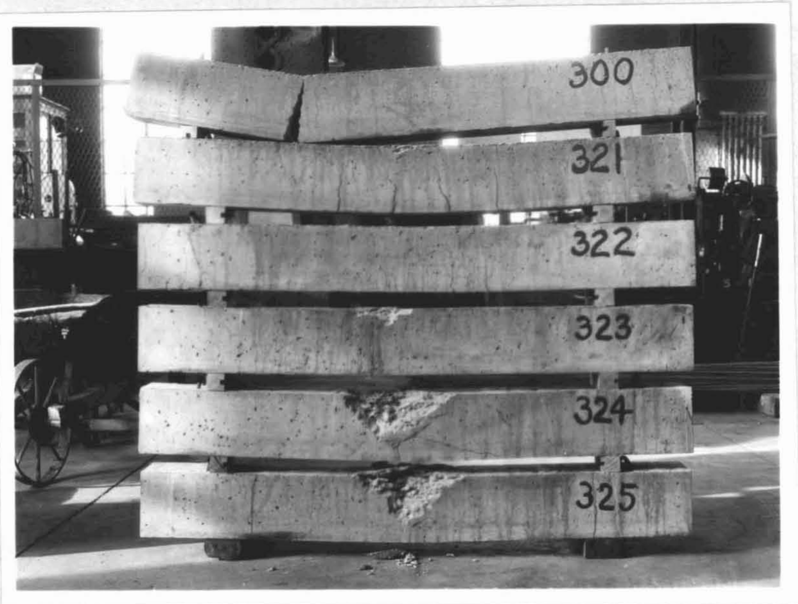


Fig. 14 - Beams Having a c/w Ratio of 1.50, After Testing

Fig. 15 - Beams Having a c/w Ratio of 1.75, After Testing

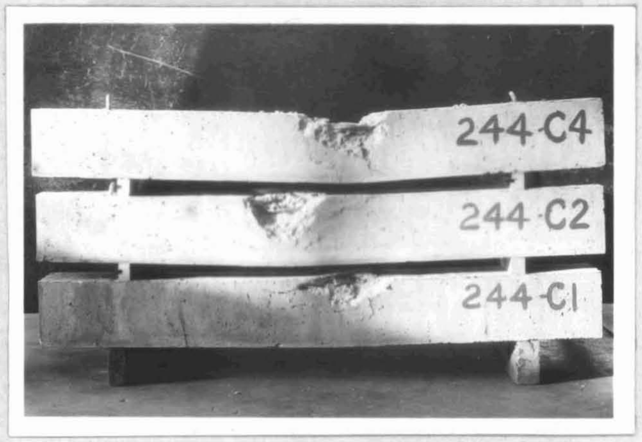


Fig. 16 - Beams With Compression Steel, After Testing

Fig. 17 - Beams of Various Depths, After Testing

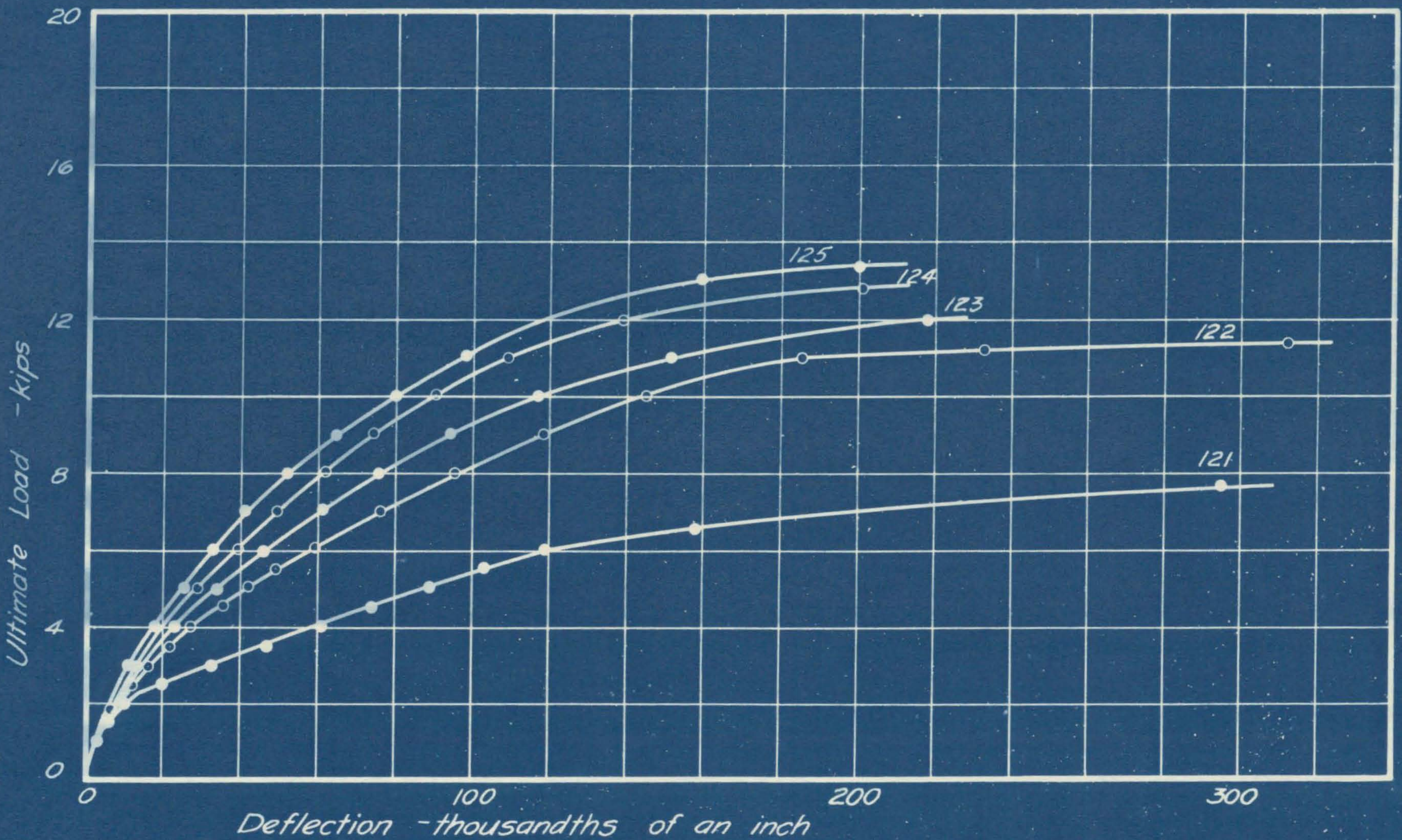


FIG. 18 -LOAD DEFLECTION CURVES FOR BEAMS 121, 122, 123, 124, and 125.

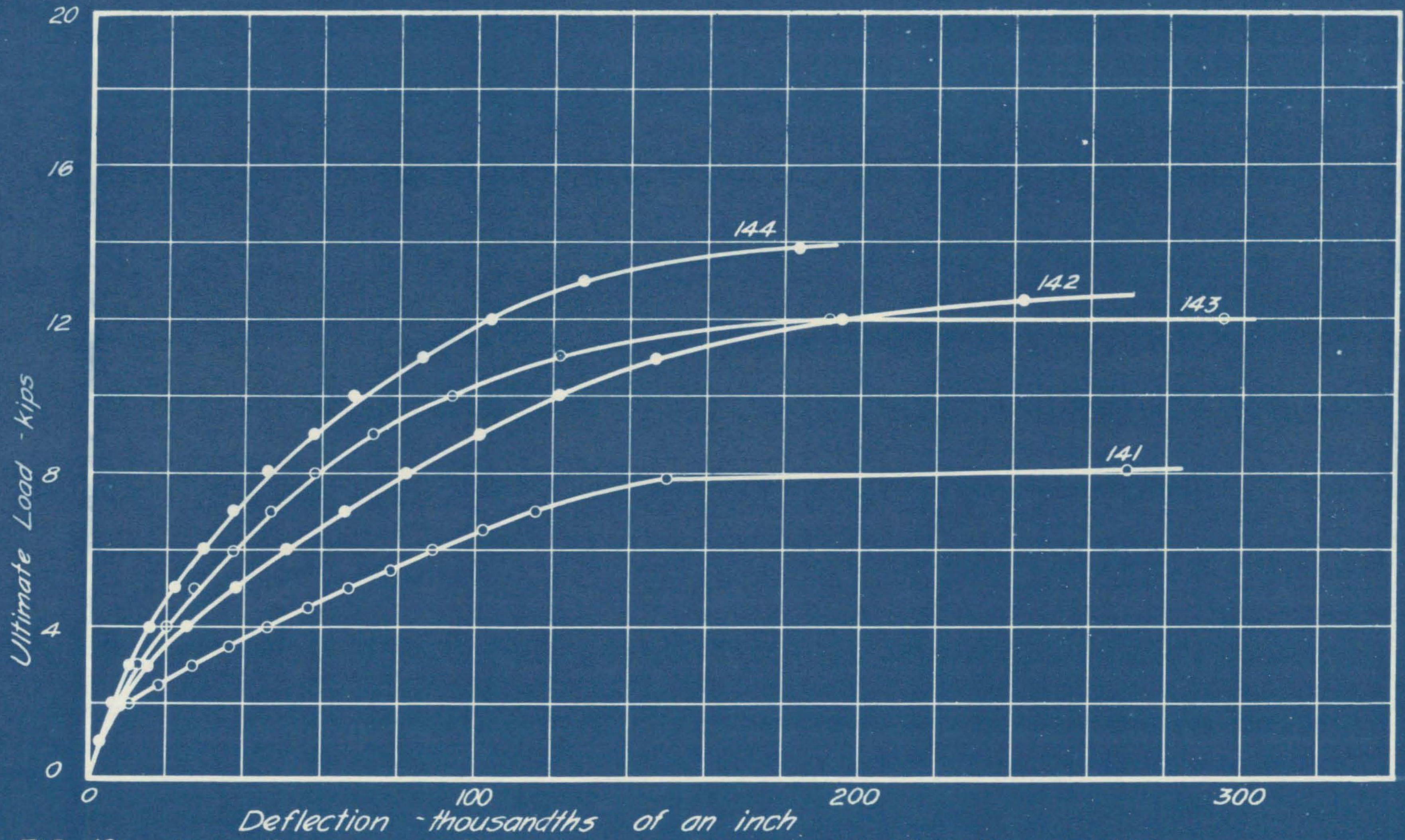


FIG. 19 - LOAD-DEFLECTION CURVES FOR BEAMS 141, 142, 143, and 144.

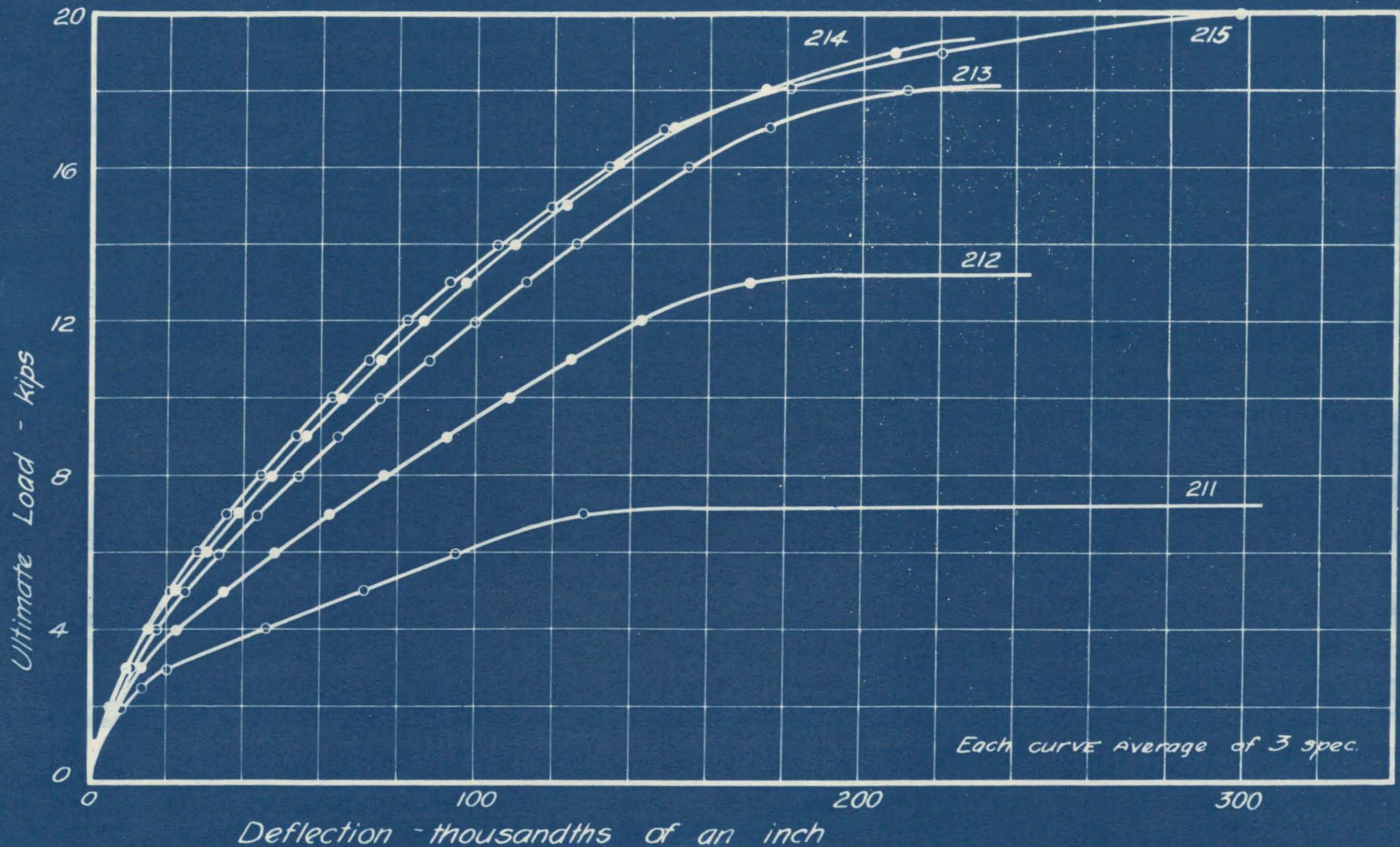


FIG. 20 - LOAD-DEFLECTION CURVES FOR BEAMS 211, 212, 213, 214, and 215.

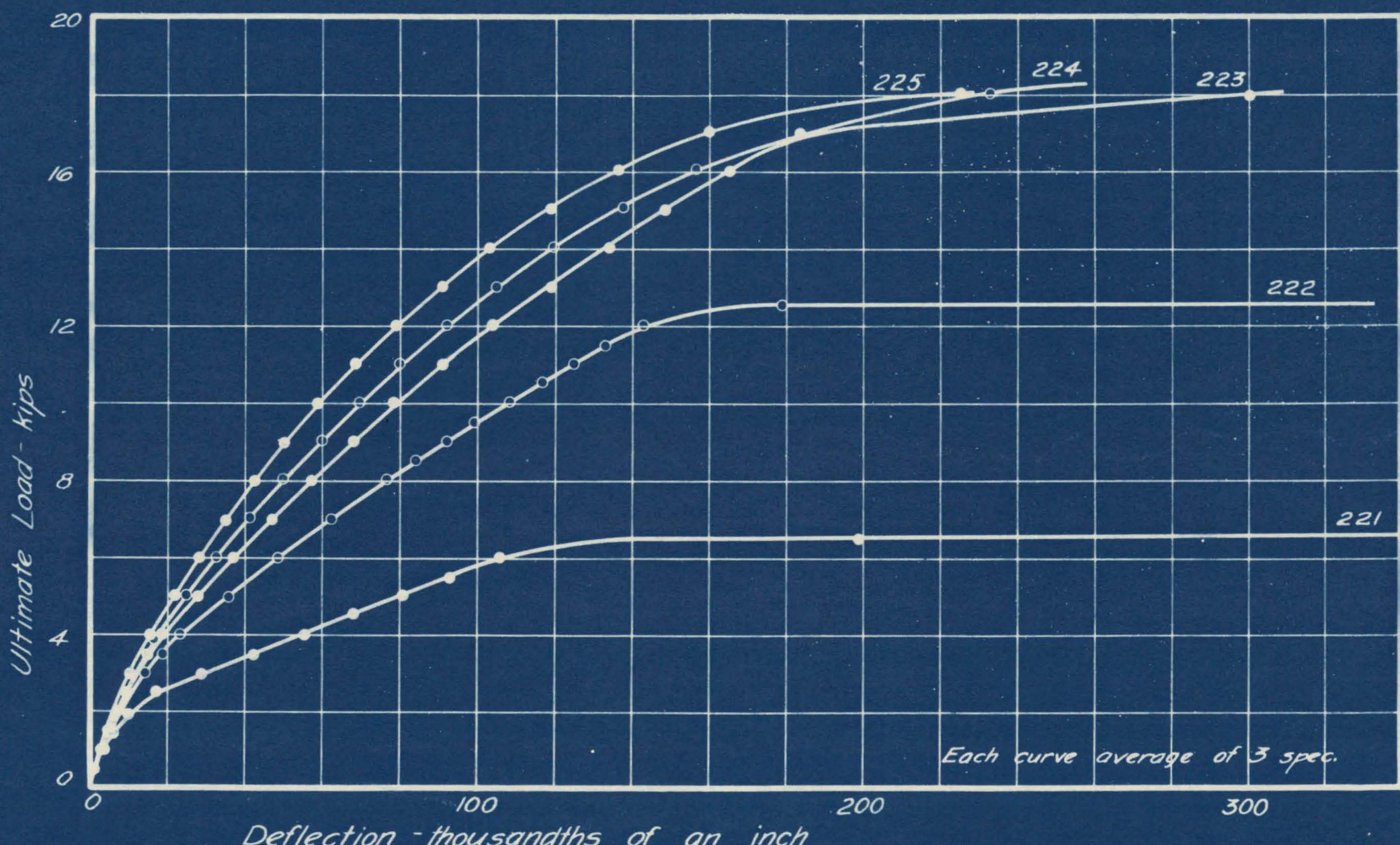


FIG. 21 - LOAD-DEFLECTION CURVES FOR BEAMS 221, 222, 223, 224, and 225.

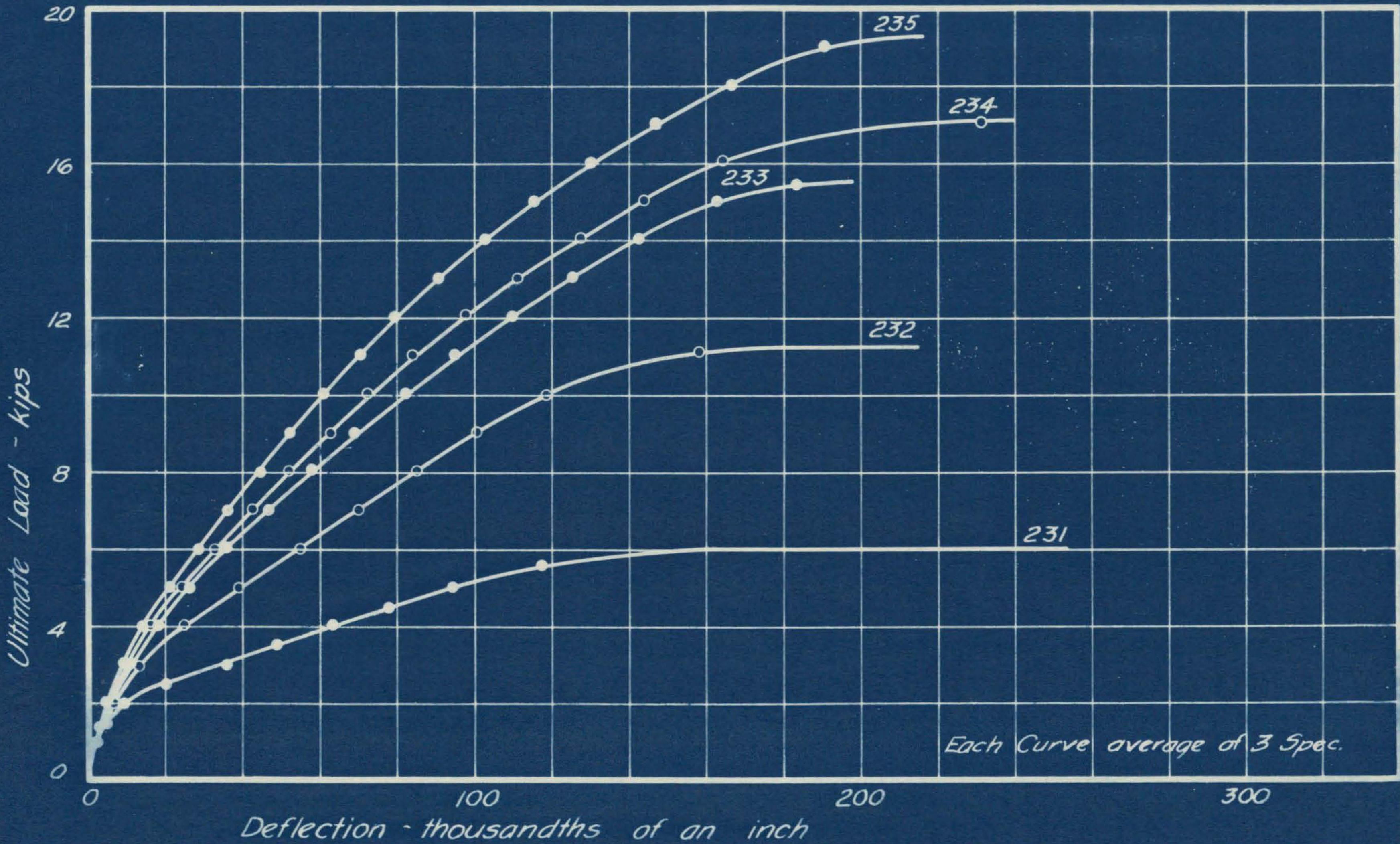


FIG. 22 - LOAD-DEFLECTION CURVES FOR BEAMS 231, 232, 233, 234, and 235.

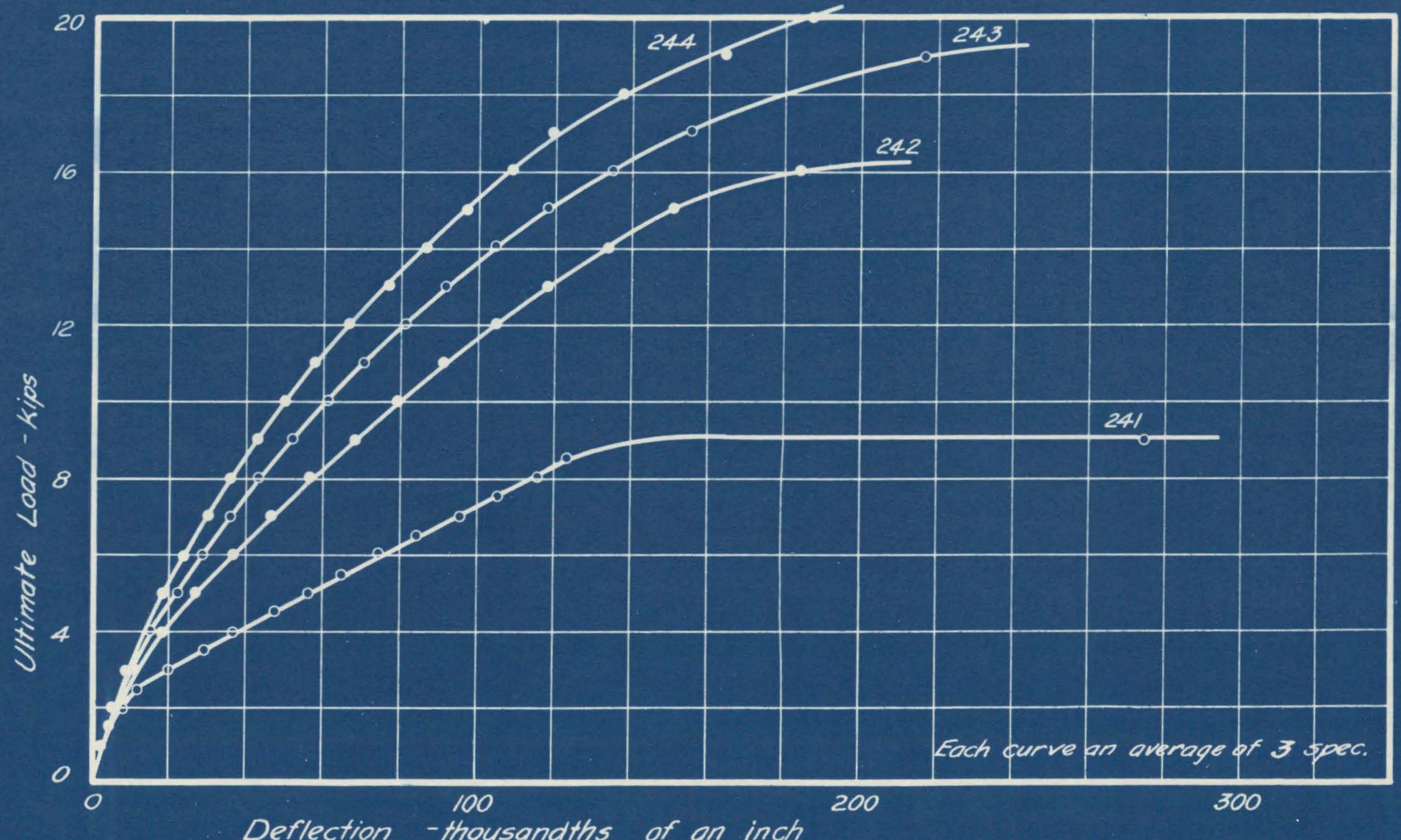


FIG. 23 - LOAD-DEFLECTION CURVES FOR BEAMS 241, 242, 243, and 244.

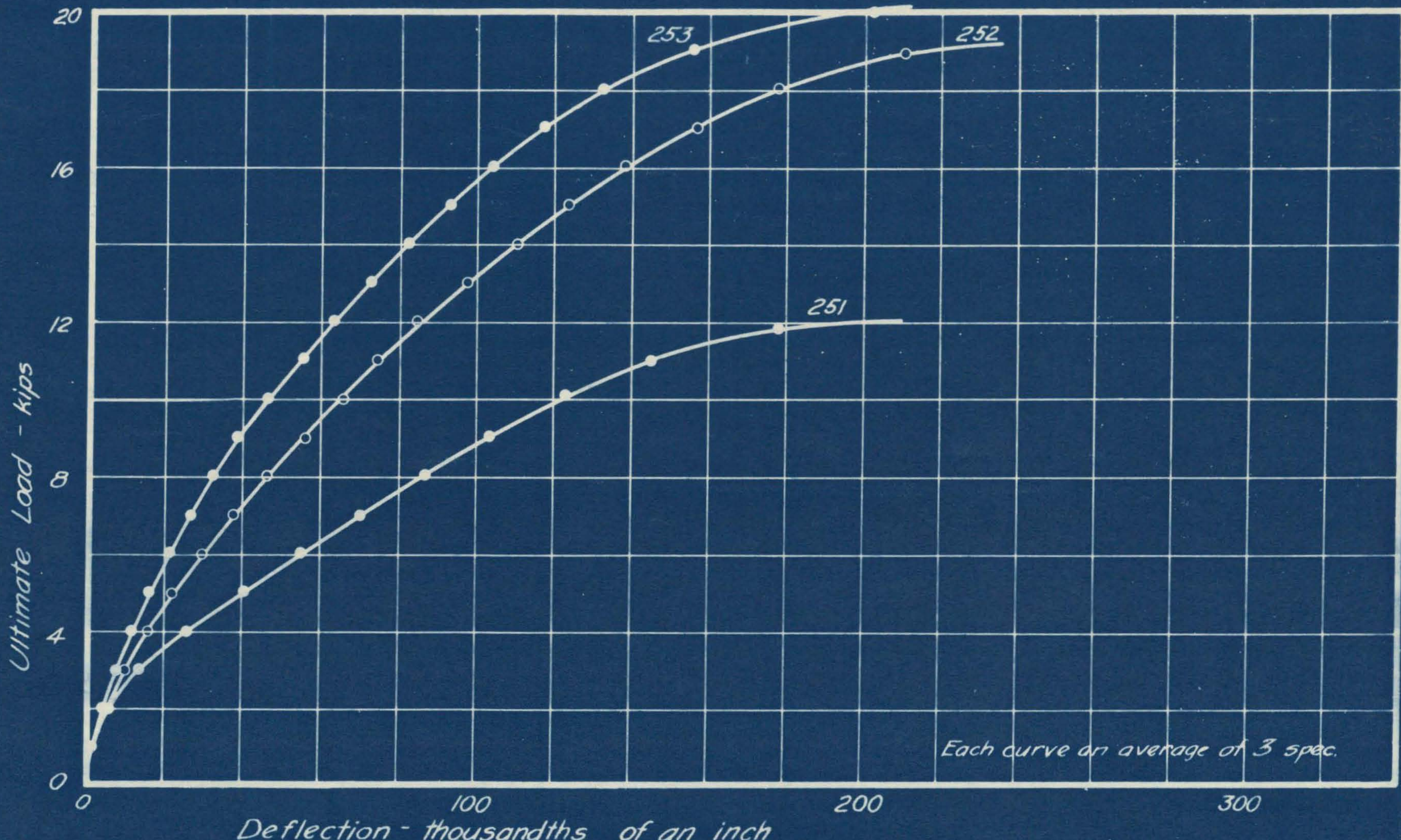


FIG. 24 - LOAD-DEFLECTION CURVES FOR BEAMS 251, 252, and 253.

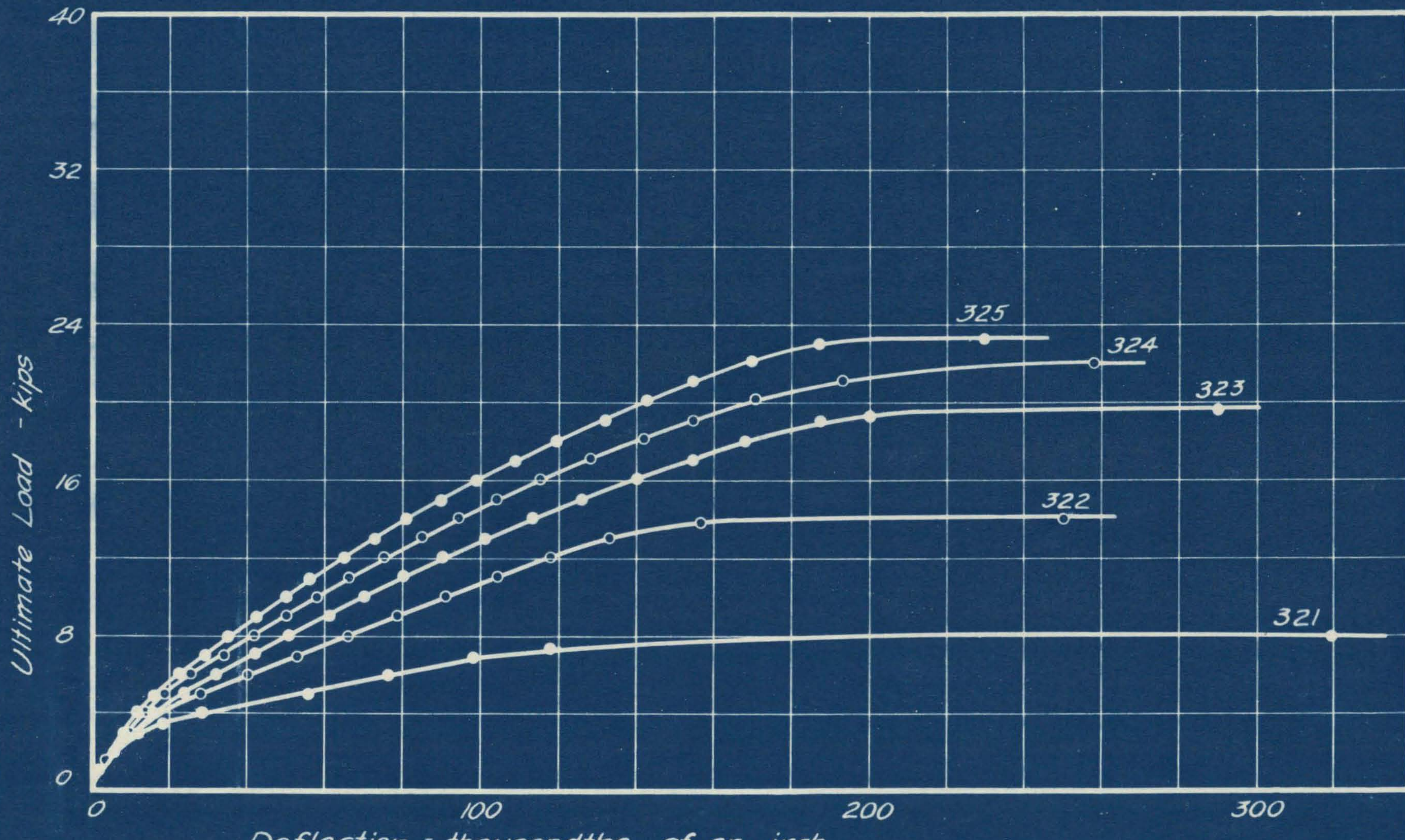


FIG. 25 - LOAD-DEFLECTION CURVES FOR BEAMS 321, 322, 323, 324, and 325.

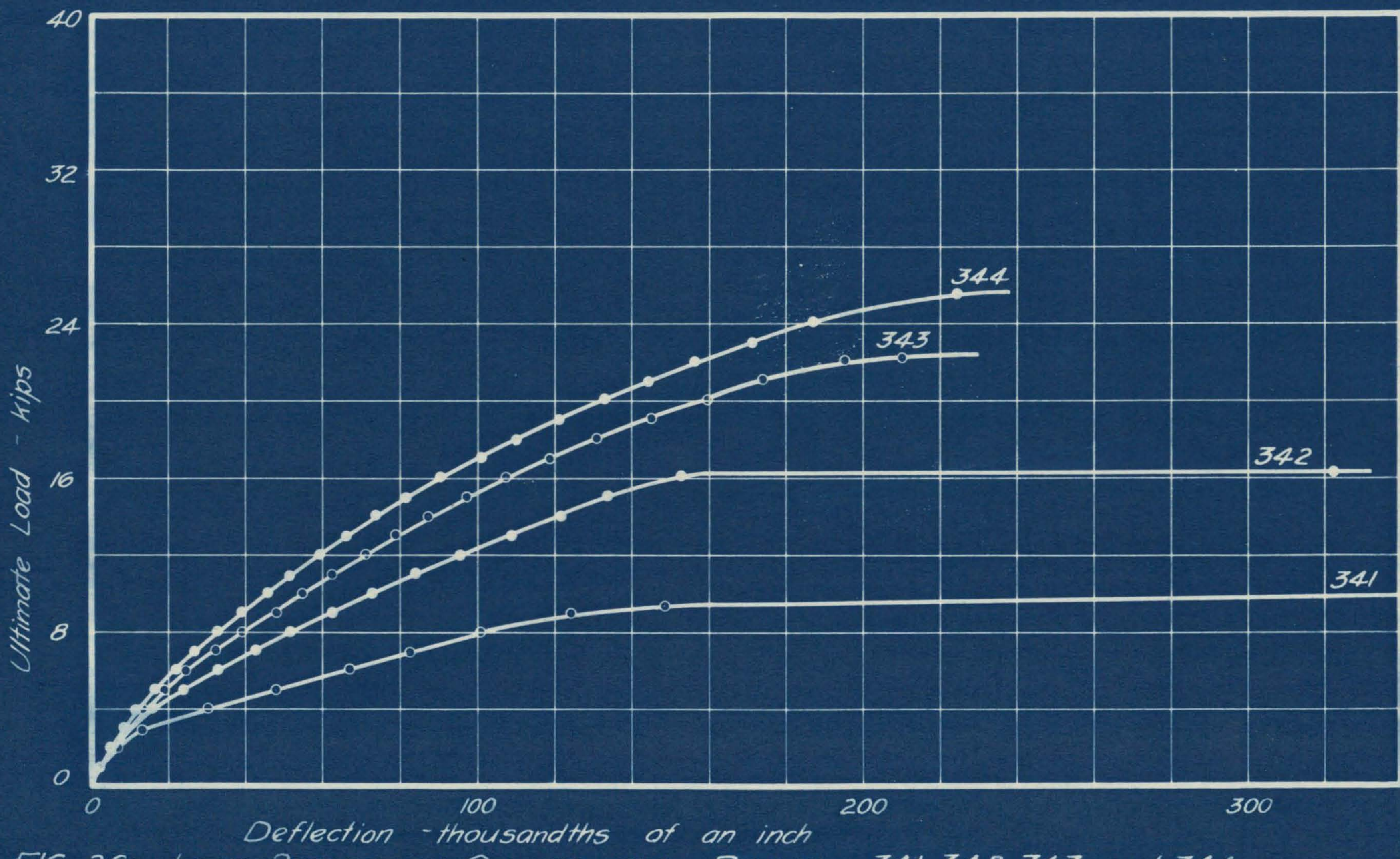


FIG. 26 - LOAD-DEFLECTION CURVES FOR BEAMS 341, 342, 343, and 344.

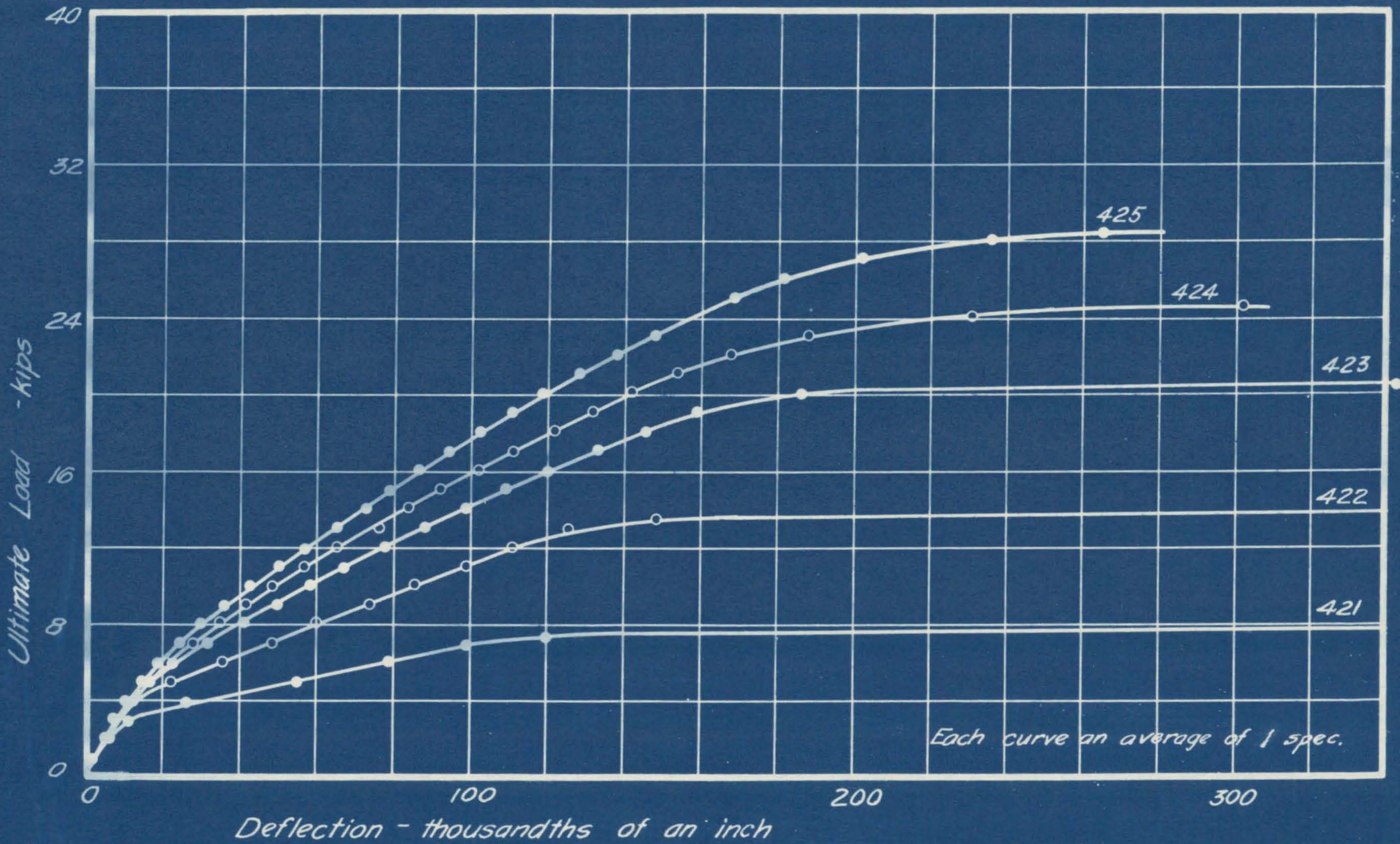


FIG. 27 - LOAD-DEFLECTION CURVES FOR BEAMS 421, 422, 423, 424, and 425.

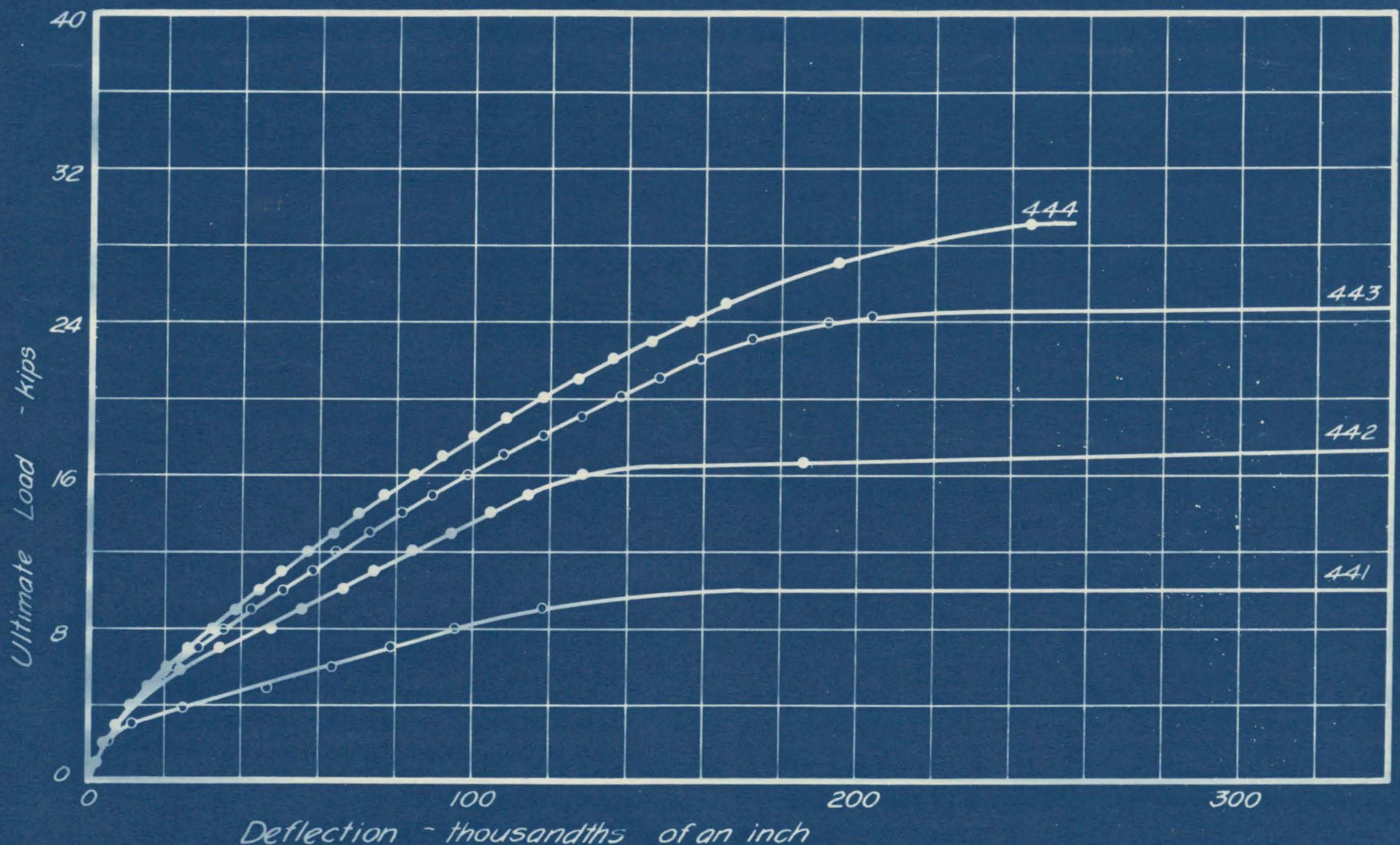


FIG. 28 - LOAD-DEFLECTION CURVES FOR BEAMS 441, 442, 443, and 444.

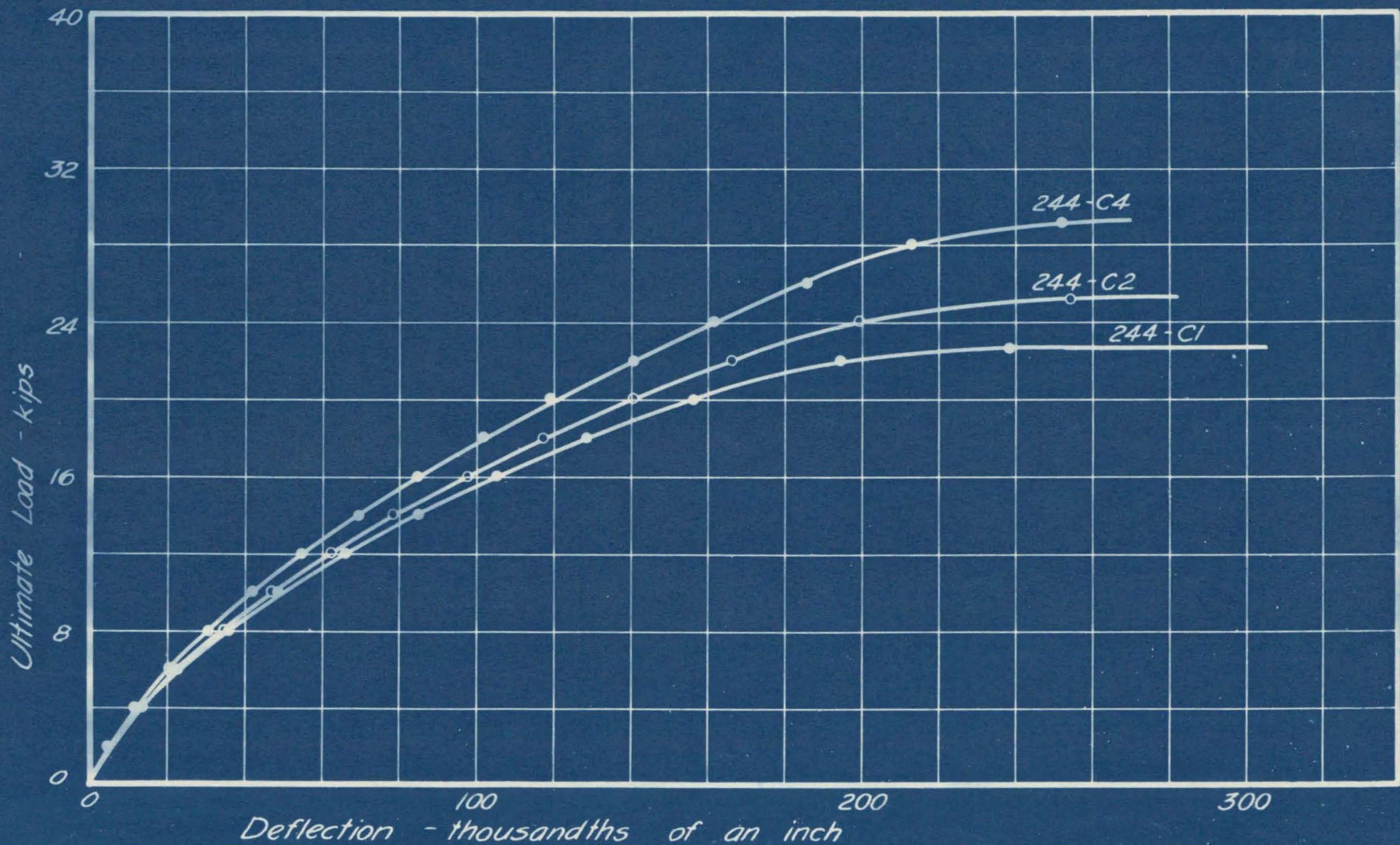


FIG. 29 - LOAD-DEFLECTION CURVES FOR BEAMS 244-C1, 244-C2, and 244-C4

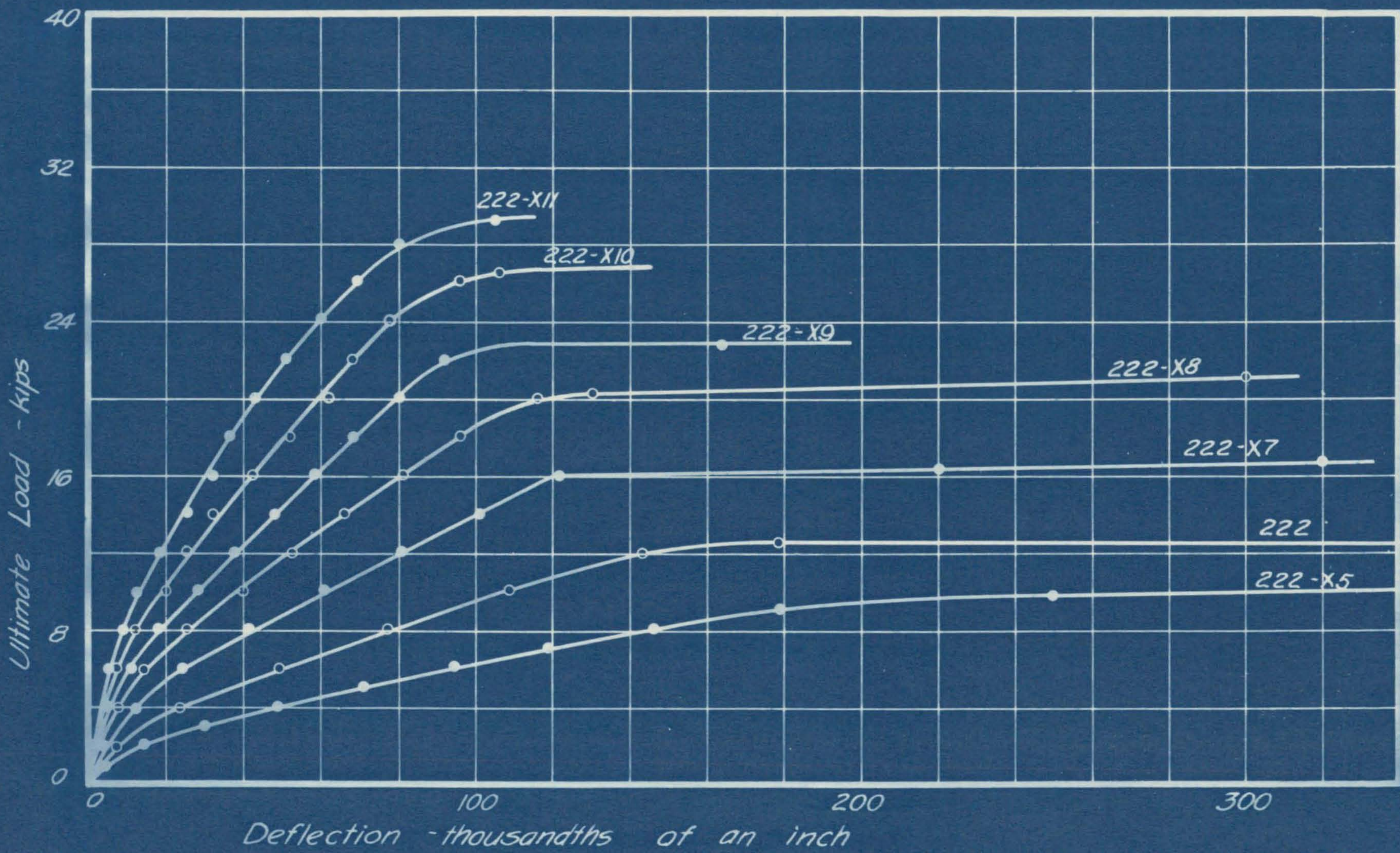


FIG. 30 - LOAD-DEFLECTION CURVES FOR BEAMS 222-X5, 222, 222-X7, 222-X8, 222-X9, 222-X10, and 222-X11.

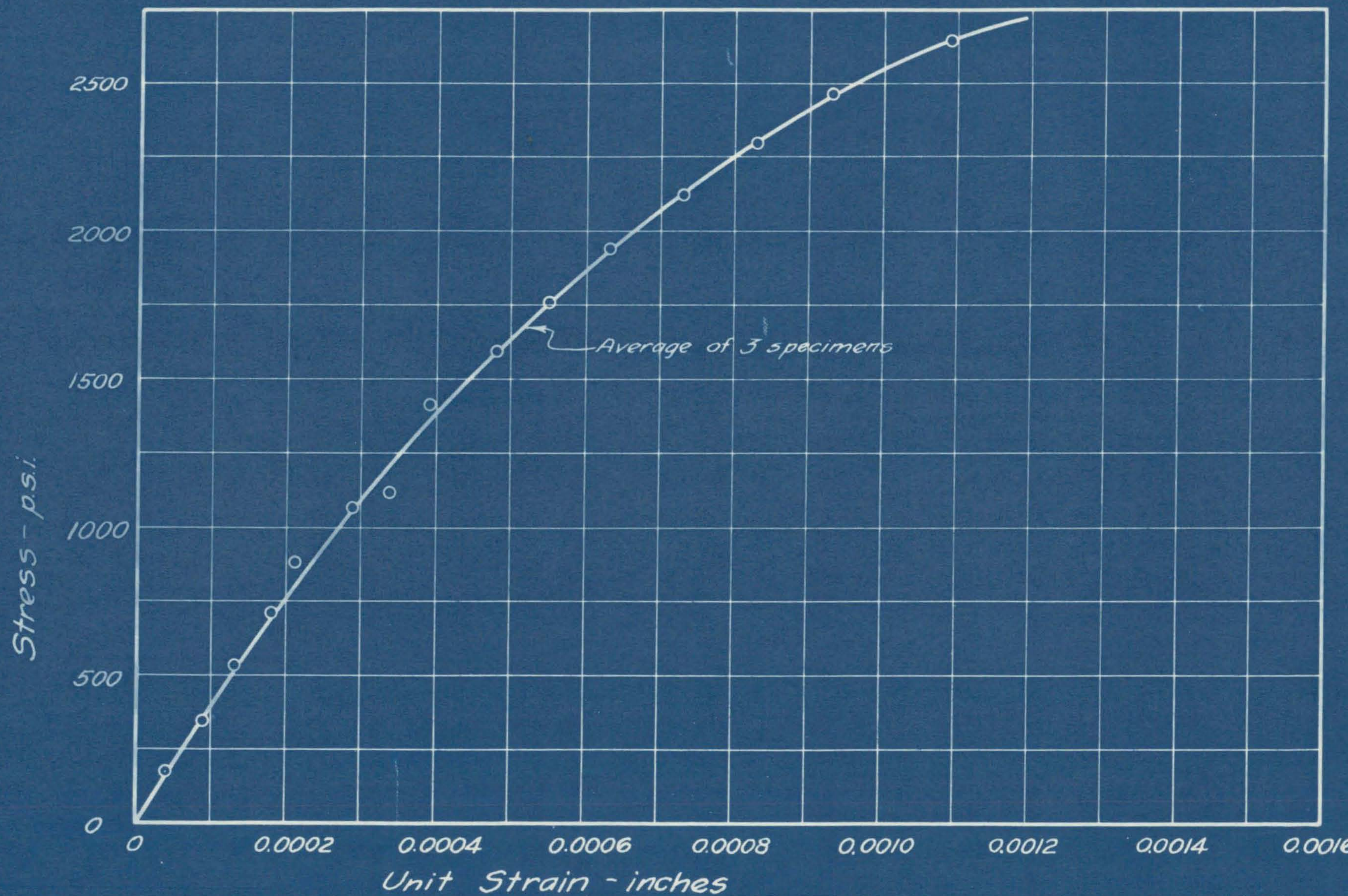


FIG. 31 - STRESS-STRAIN DIAGRAM FOR 6"×12" COMPRESSION CYLINDERS, $c/w = 1.25$

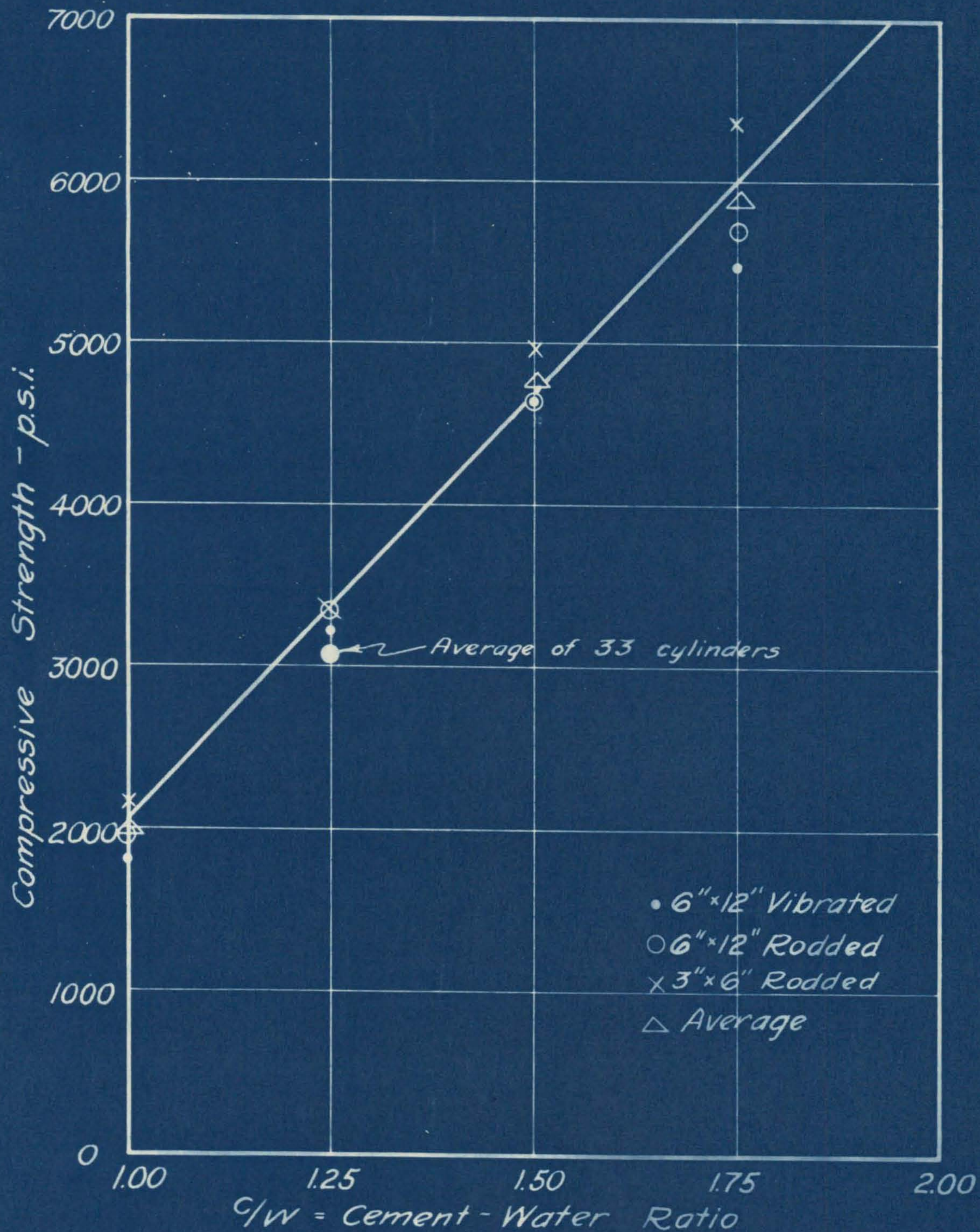


FIG. 32 - RELATION BETWEEN COMPRESSIVE STRENGTH AND CEMENT-WATER RATIO

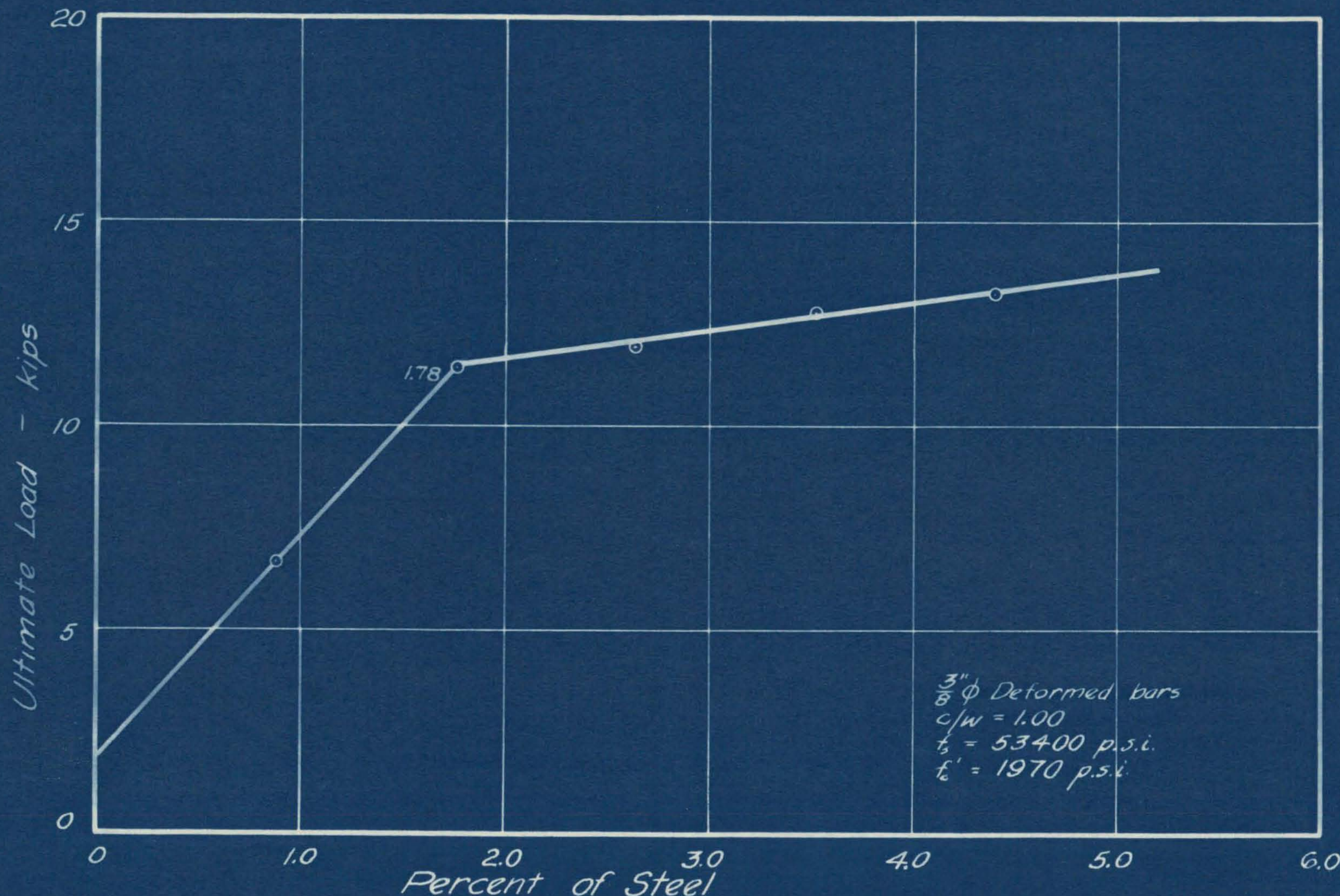


FIG. 33 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

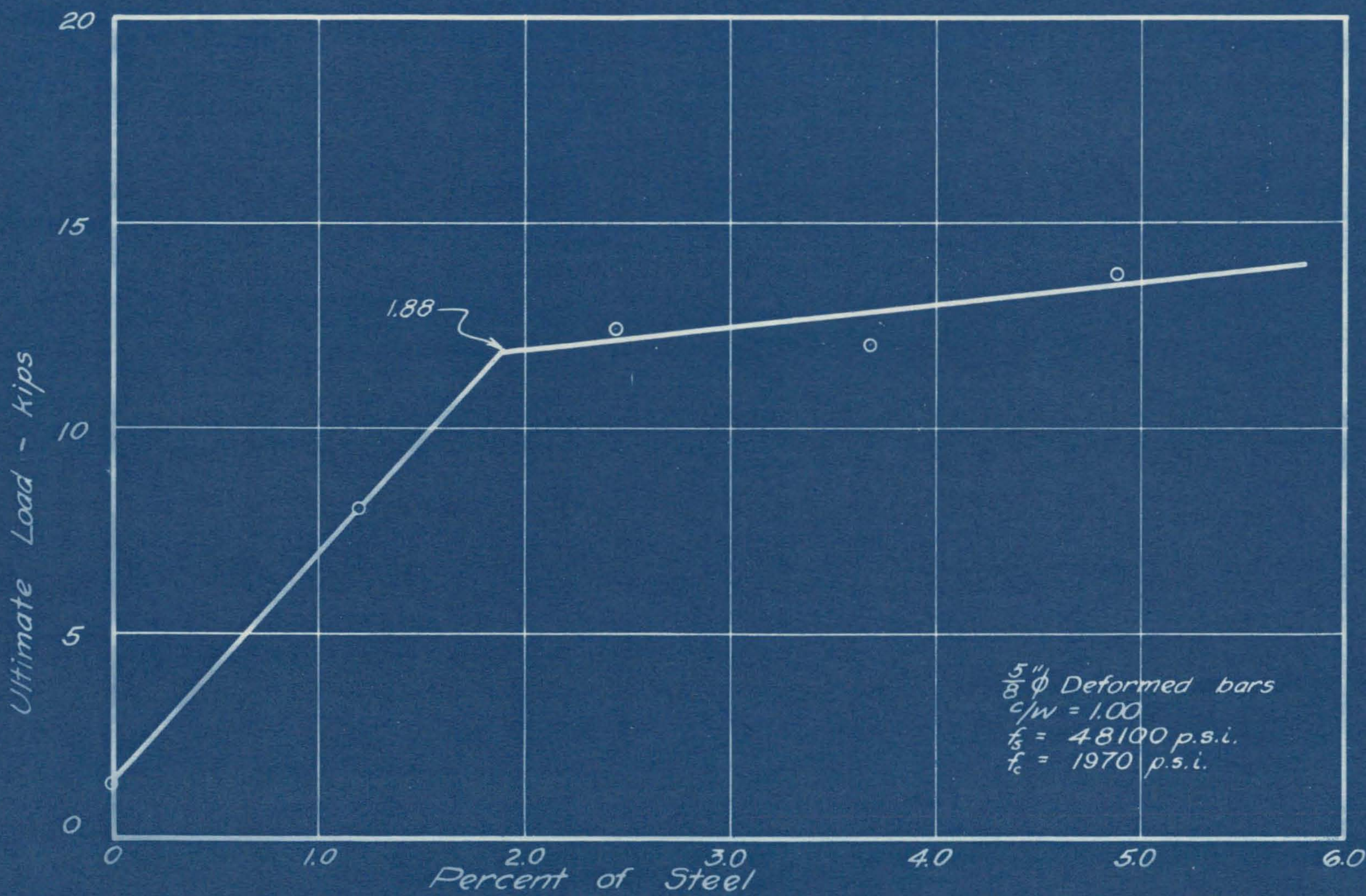


FIG. 34 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

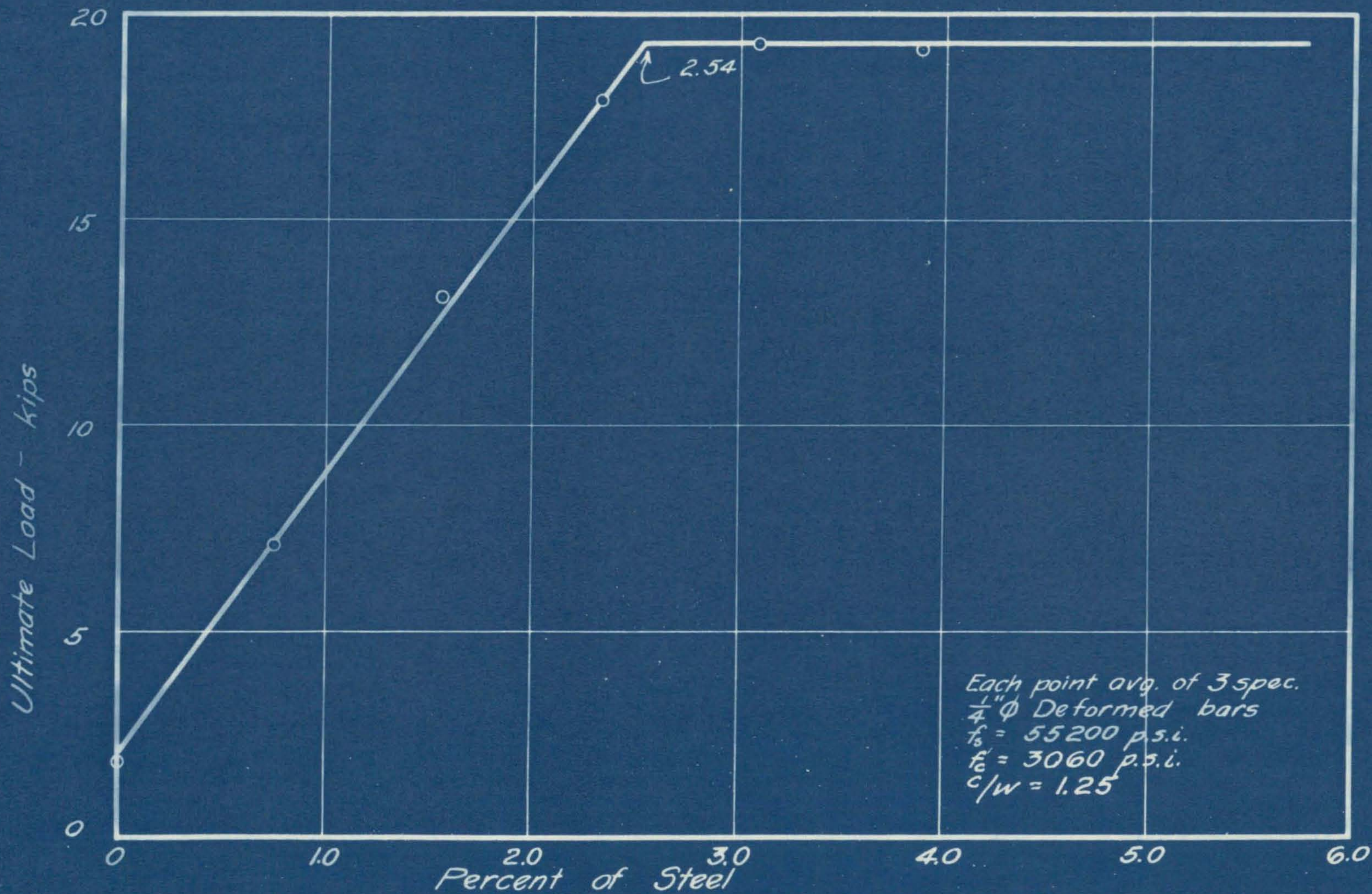


FIG. 35 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

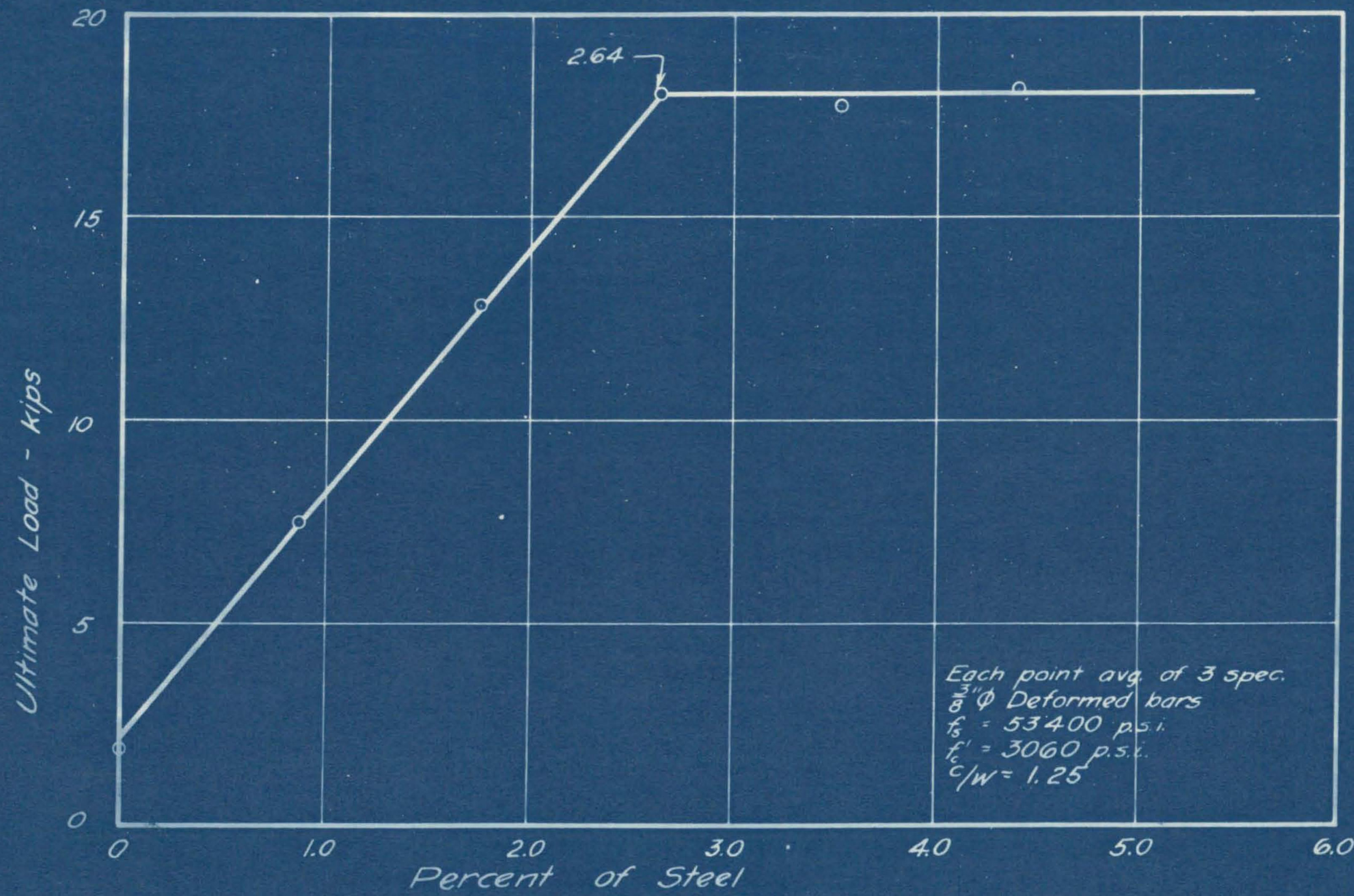


FIG. 36 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

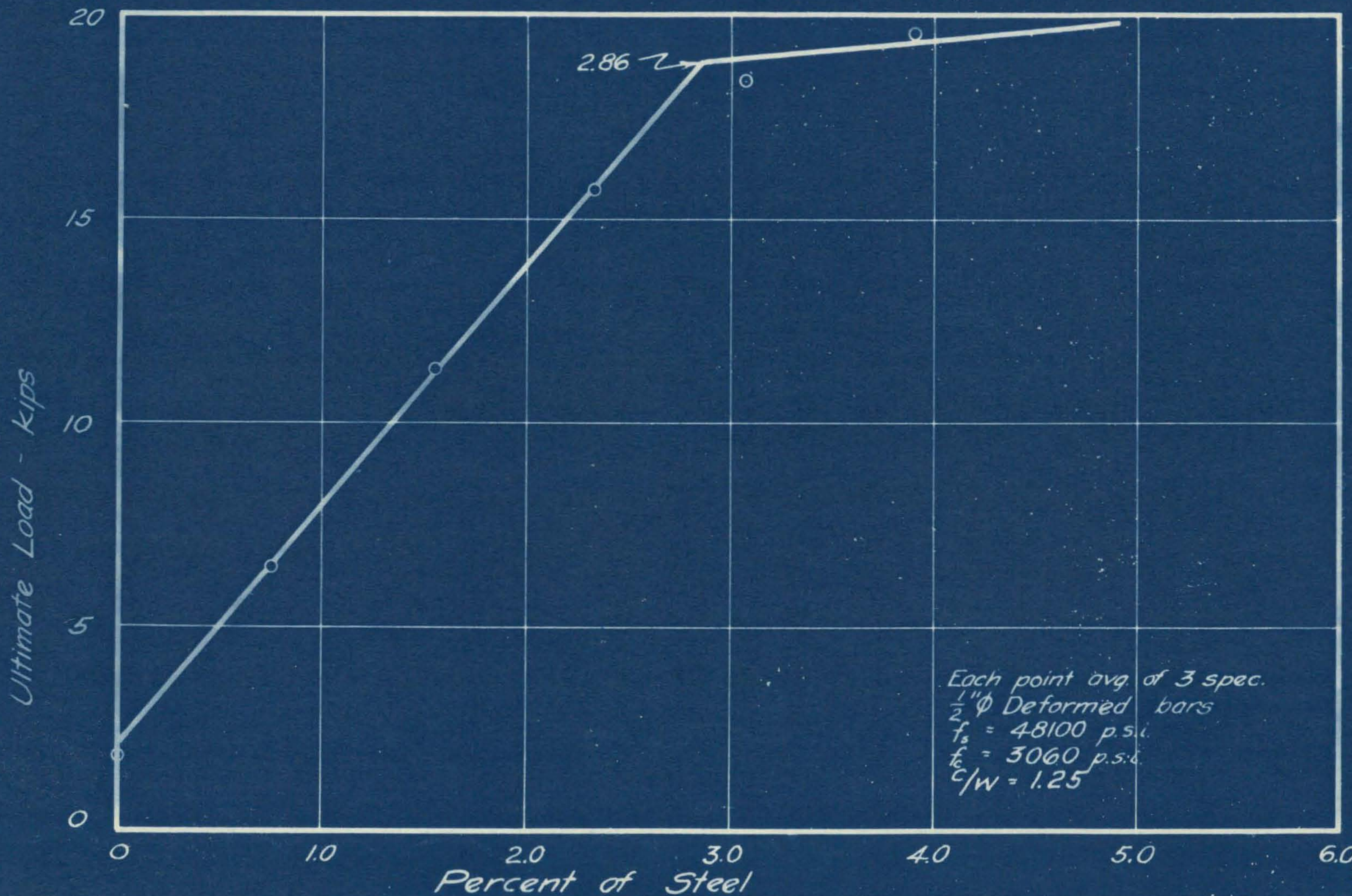


FIG. 37 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

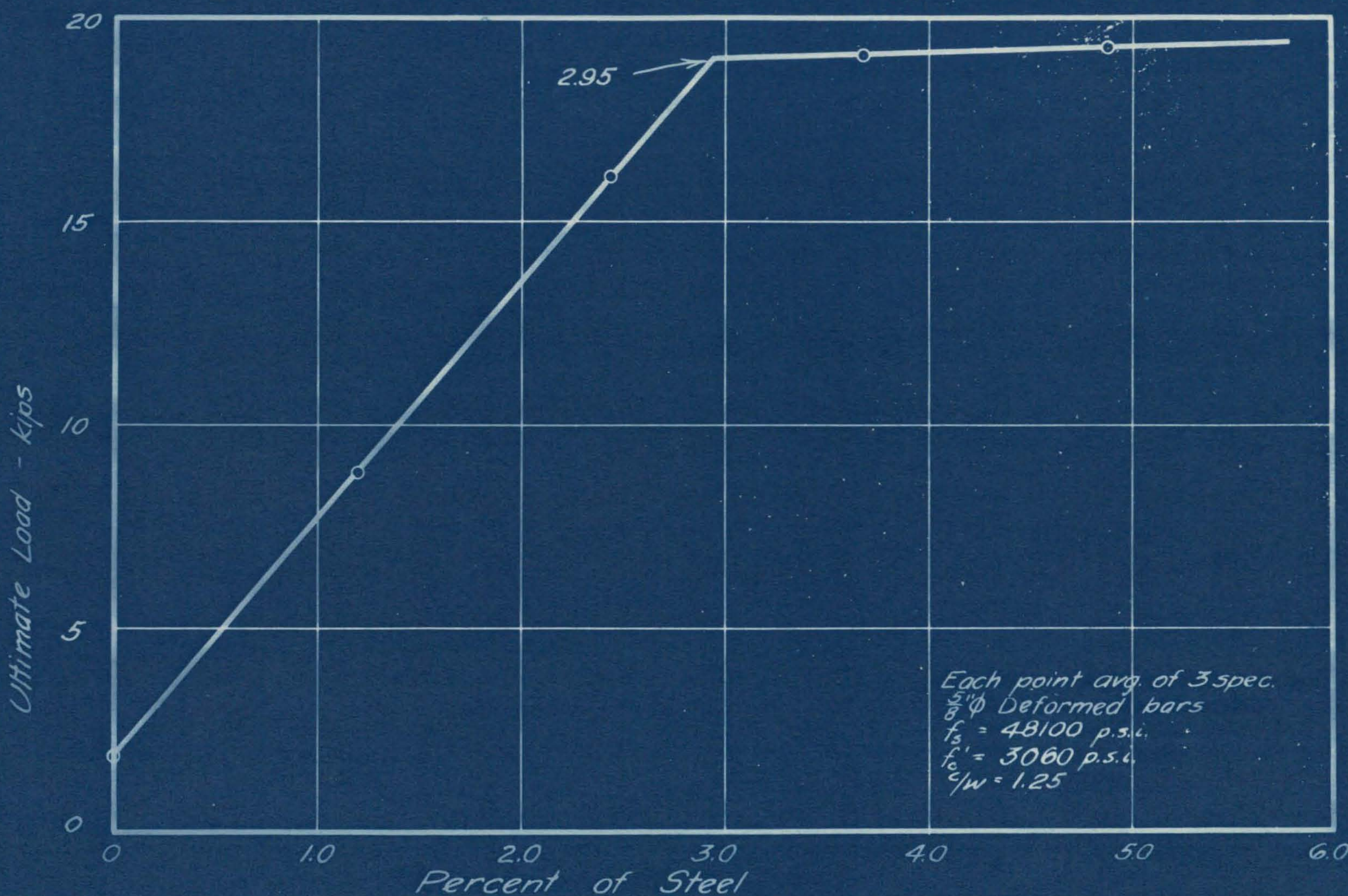


FIG. 38 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

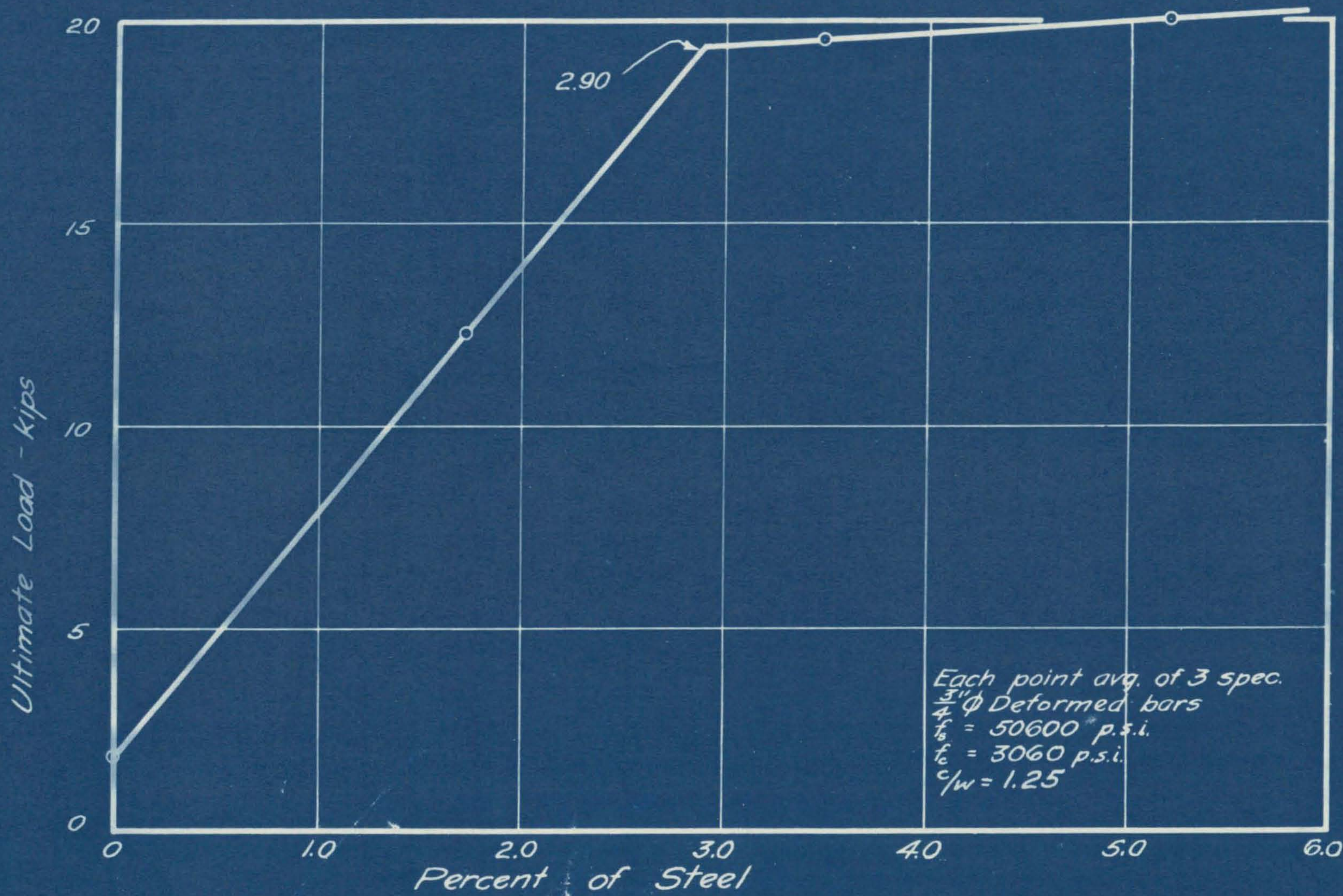


FIG. 39 - EFFECT OF PERCENT OF STEEL ON ULTIMATE LOAD

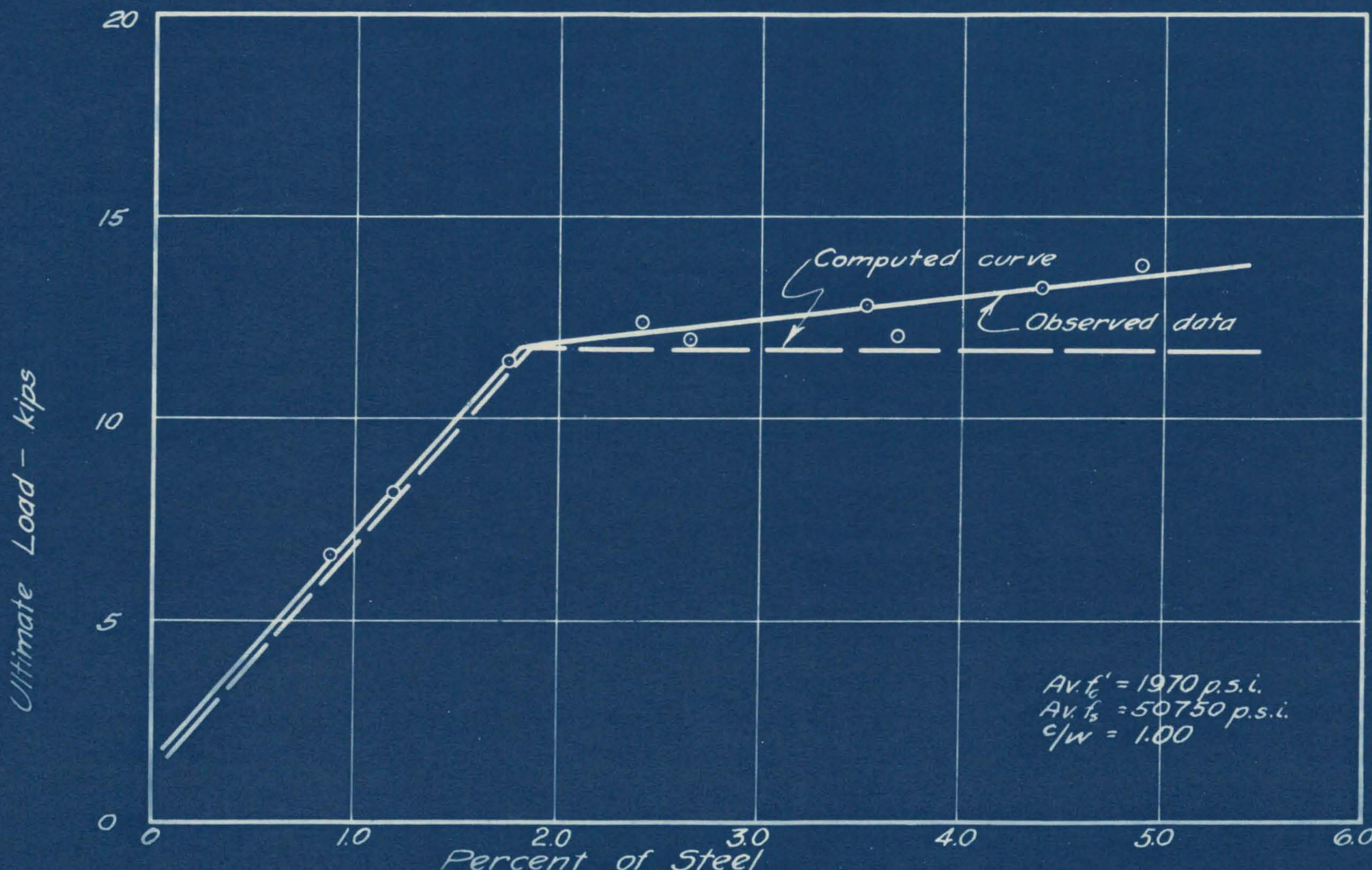


FIG. 40 - CURVE SHOWING THE EFFECT OF THE PERCENTAGE OF STEEL ON THE ULTIMATE LOAD

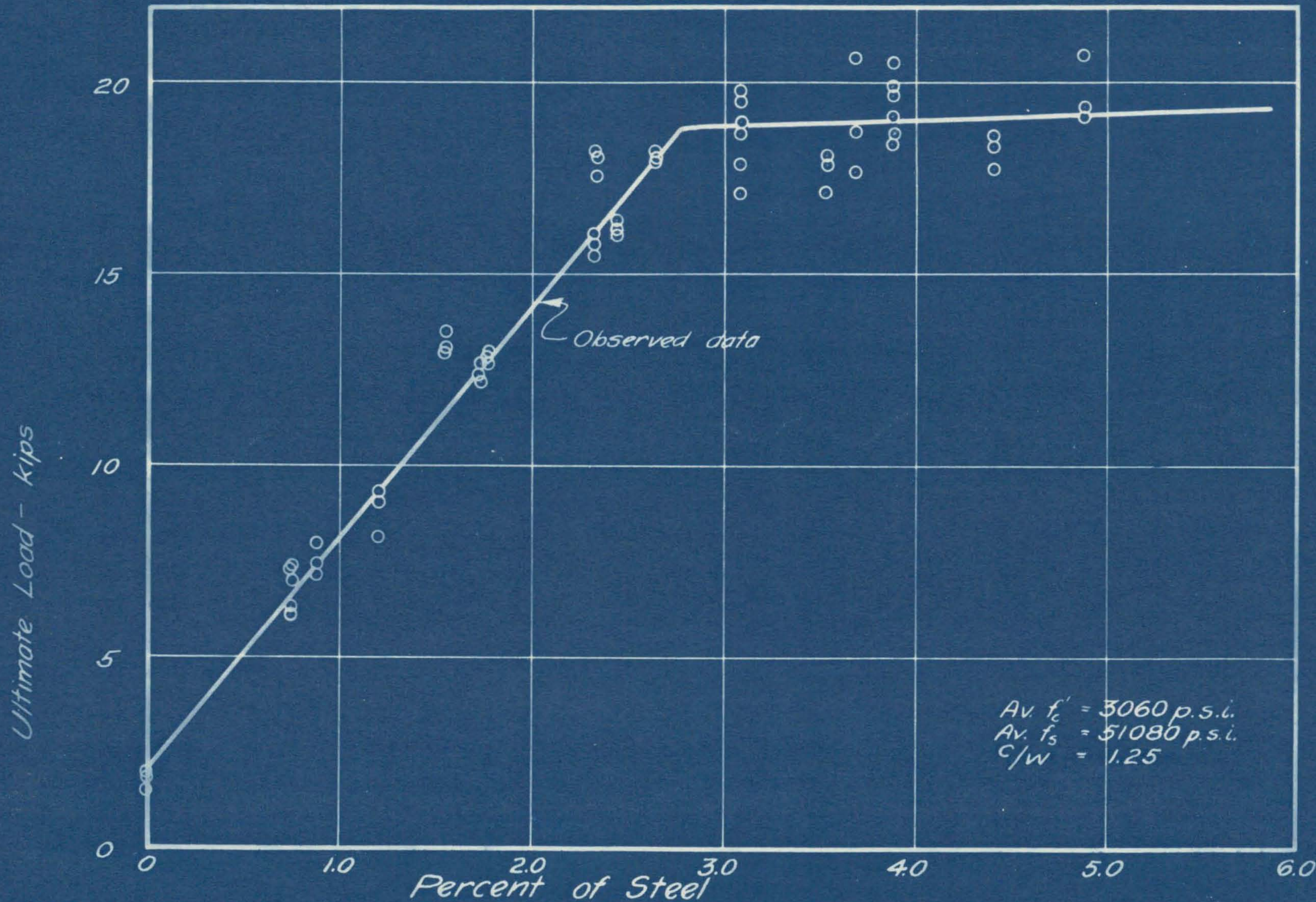


FIG. 41 - CURVE SHOWING THE EFFECT OF THE PERCENTAGE OF STEEL ON THE ULTIMATE LOAD

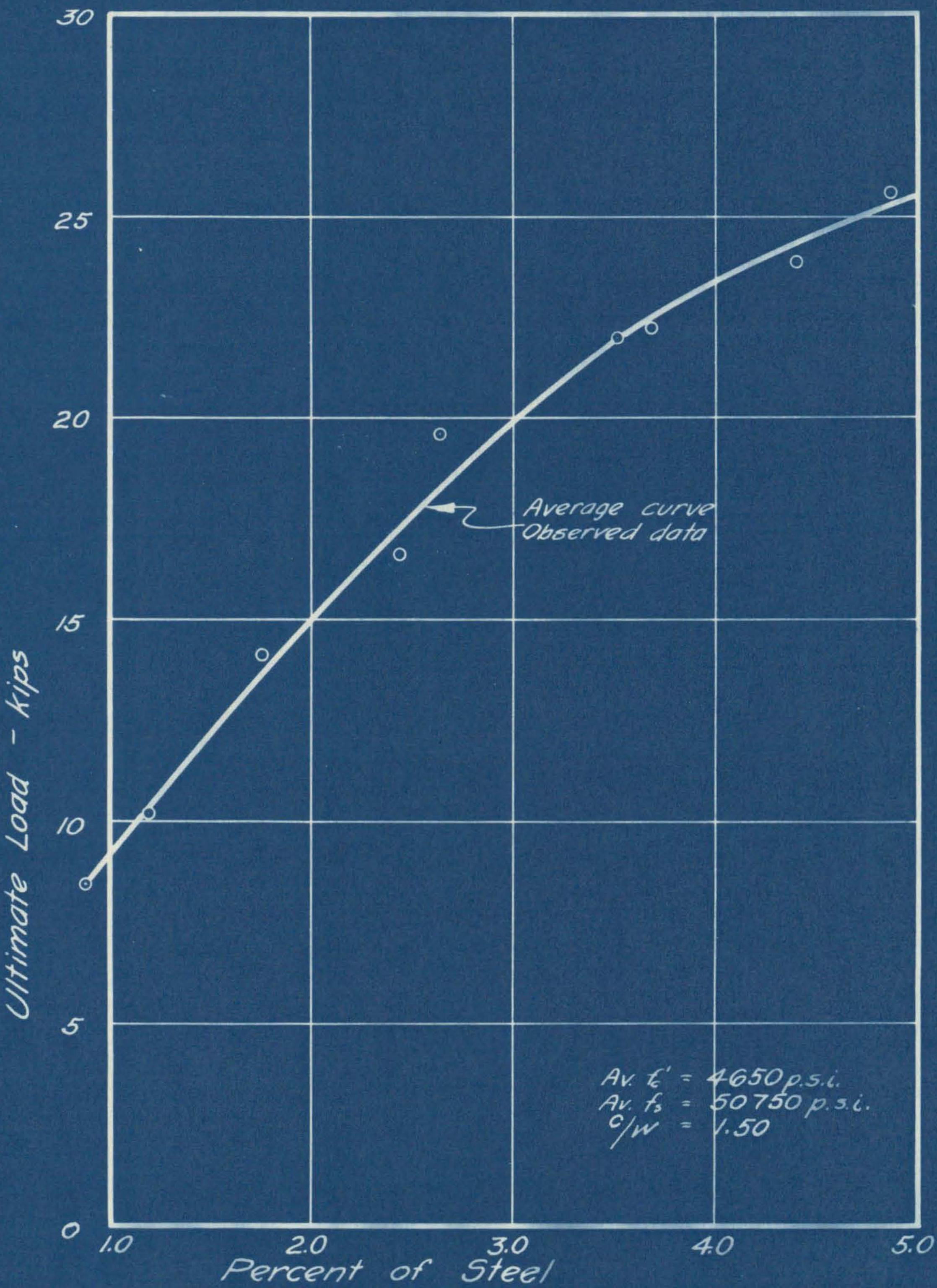


FIG. 42 - CURVE SHOWING THE EFFECT OF THE PERCENTAGE OF STEEL ON THE ULTIMATE LOAD

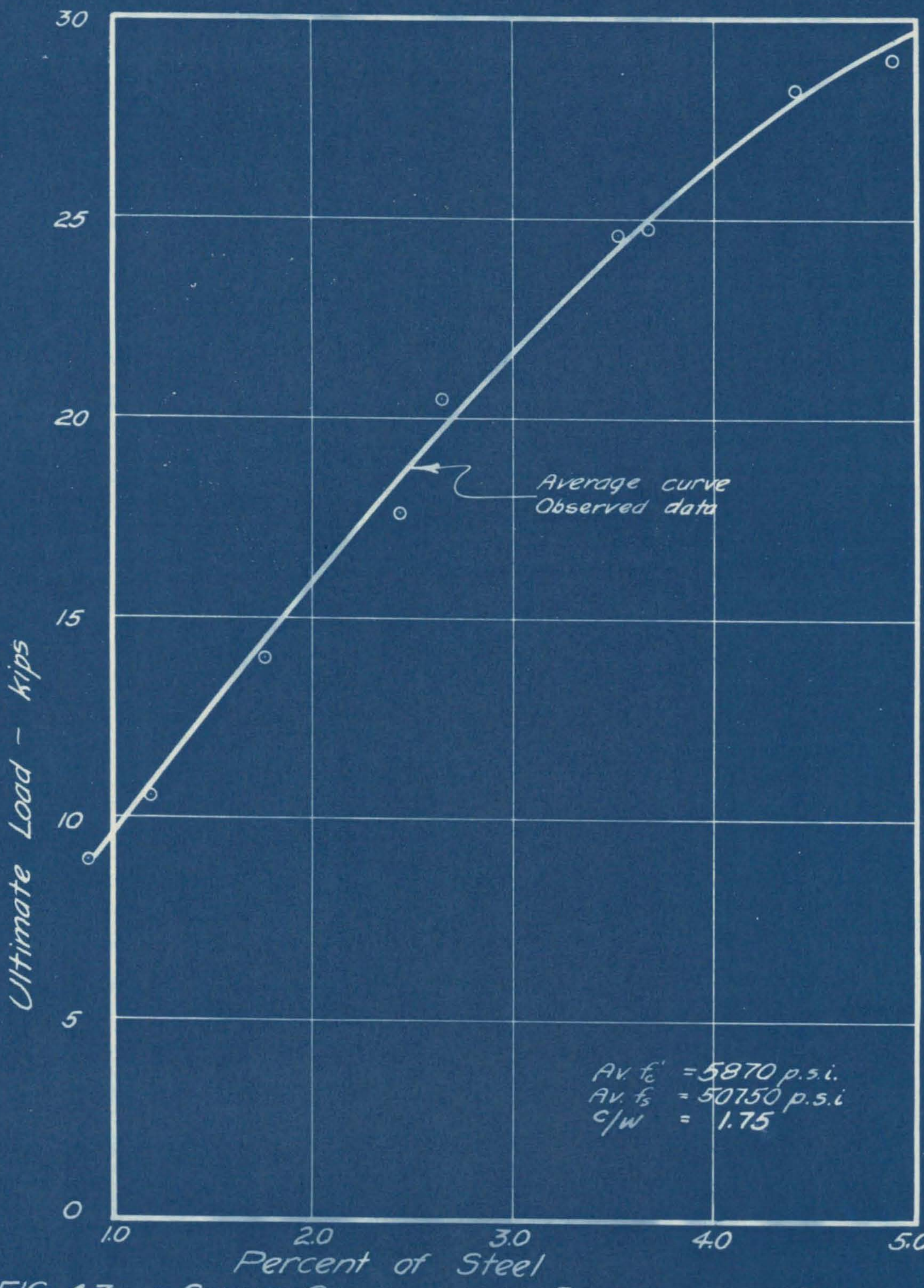


FIG. 4.3 - CURVE SHOWING THE EFFECT OF THE PERCENTAGE OF STEEL ON THE ULTIMATE LOAD

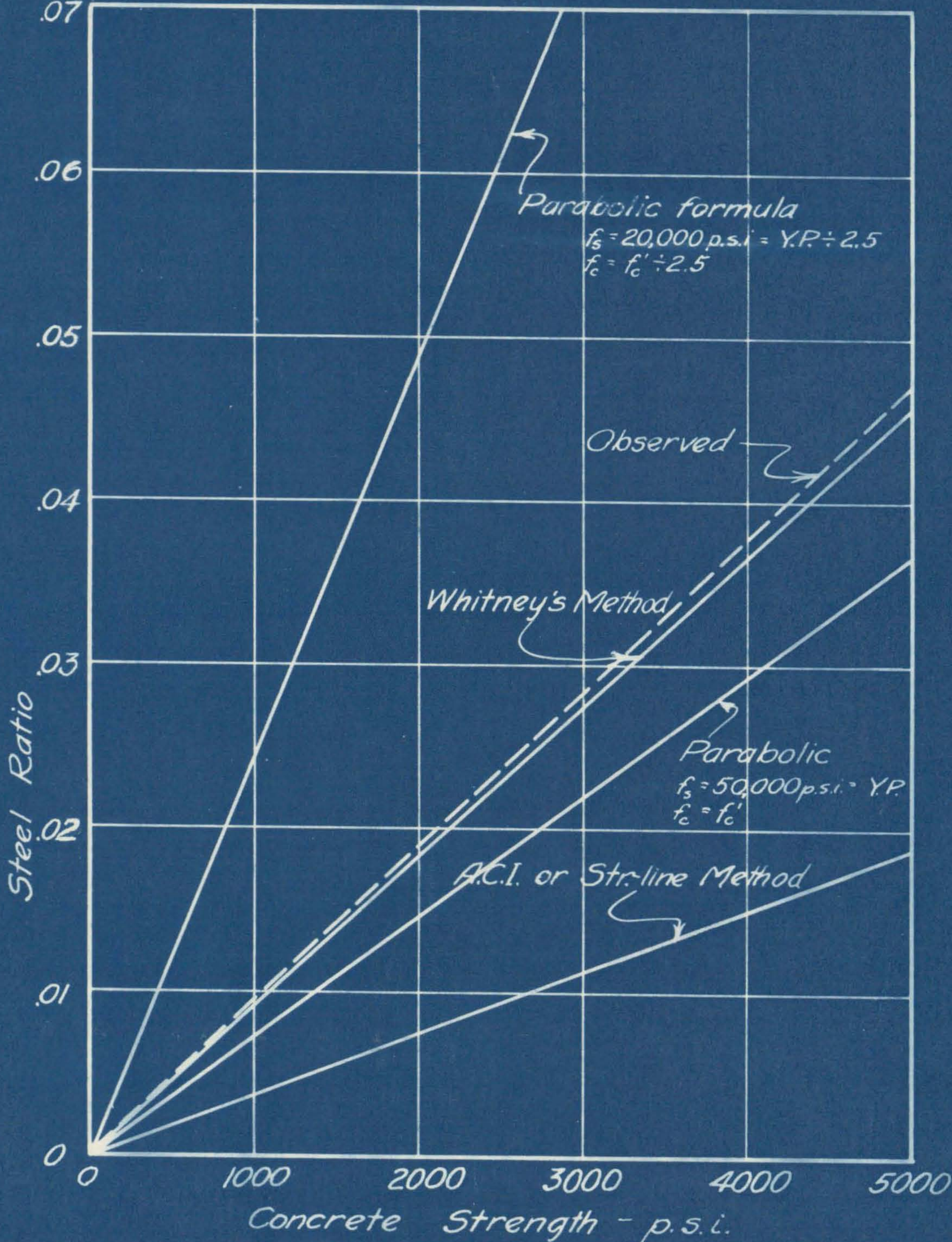


FIG. 44 - A COMPARISON OF THE STEEL RATIOS ALLOWED BY VARIOUS DESIGN METHODS FOR DIFFERENT CONCRETE STRENGTHS

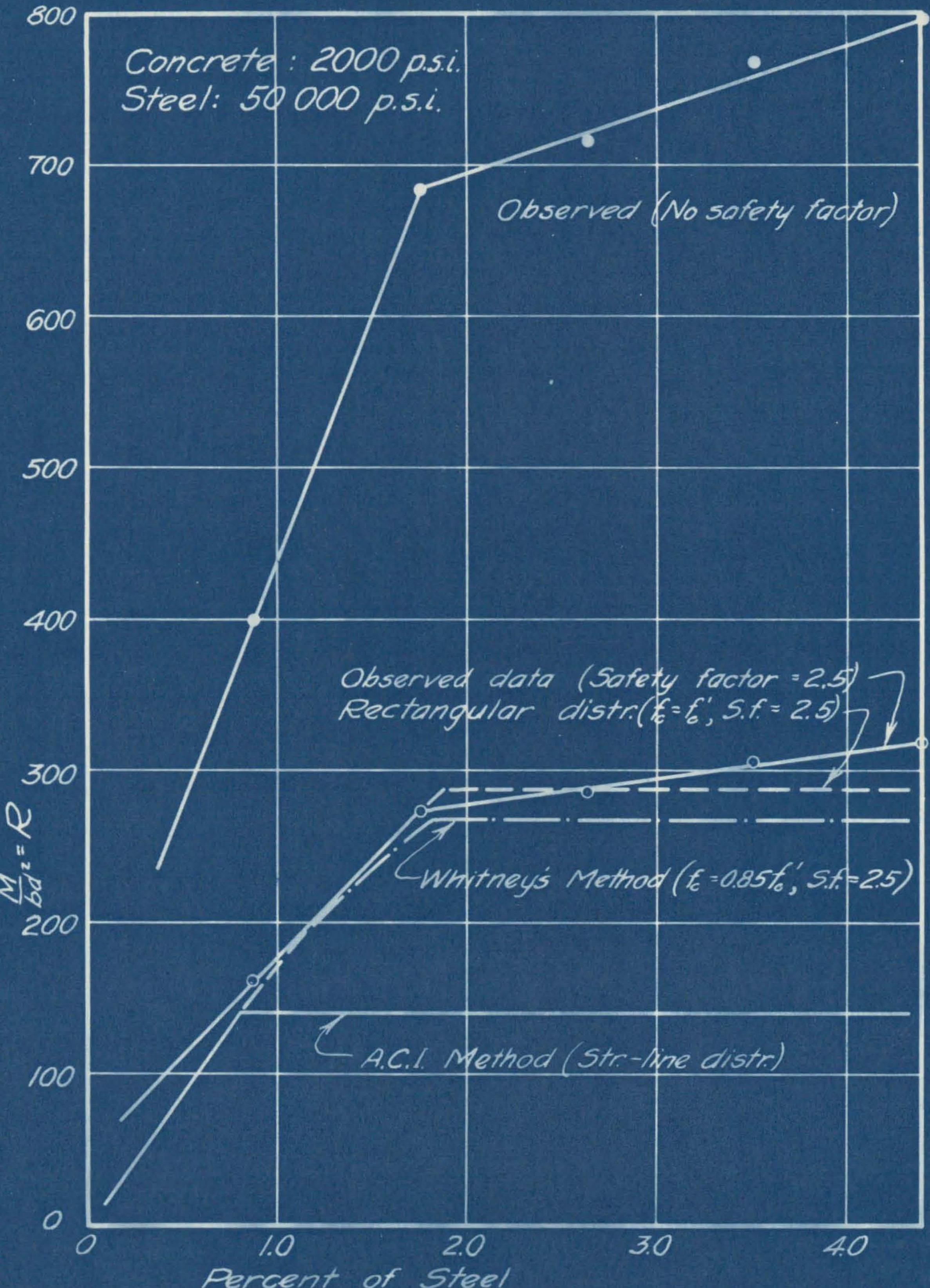


FIG. 45 - A COMPARISON OF OBSERVED AND COMPUTED "R" VALUES

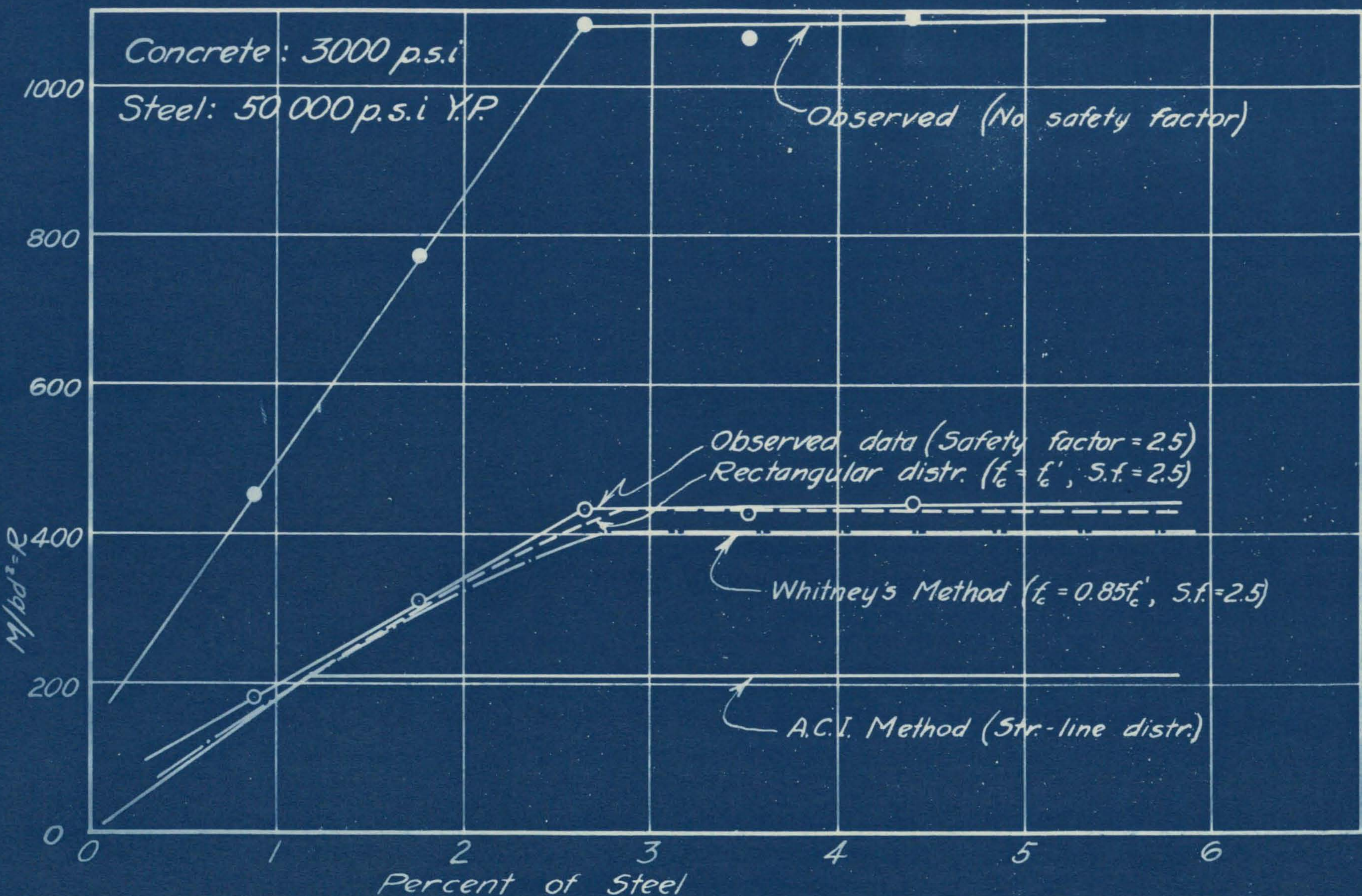


FIG. 46 – A COMPARISON OF OBSERVED AND COMPUTED "R" VALUES

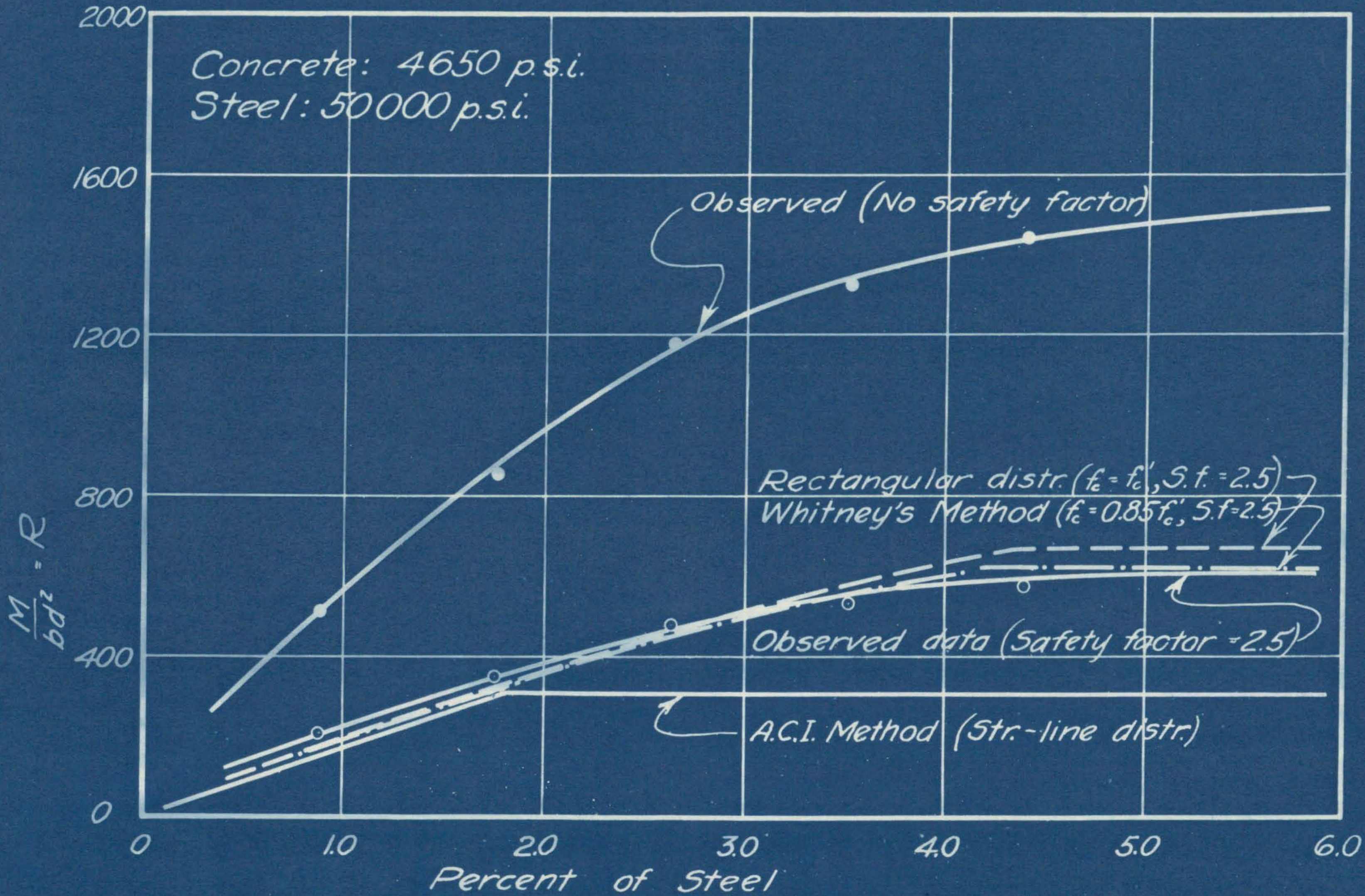


FIG. 47 – A COMPARISON OF OBSERVED AND COMPUTED "R" VALUES

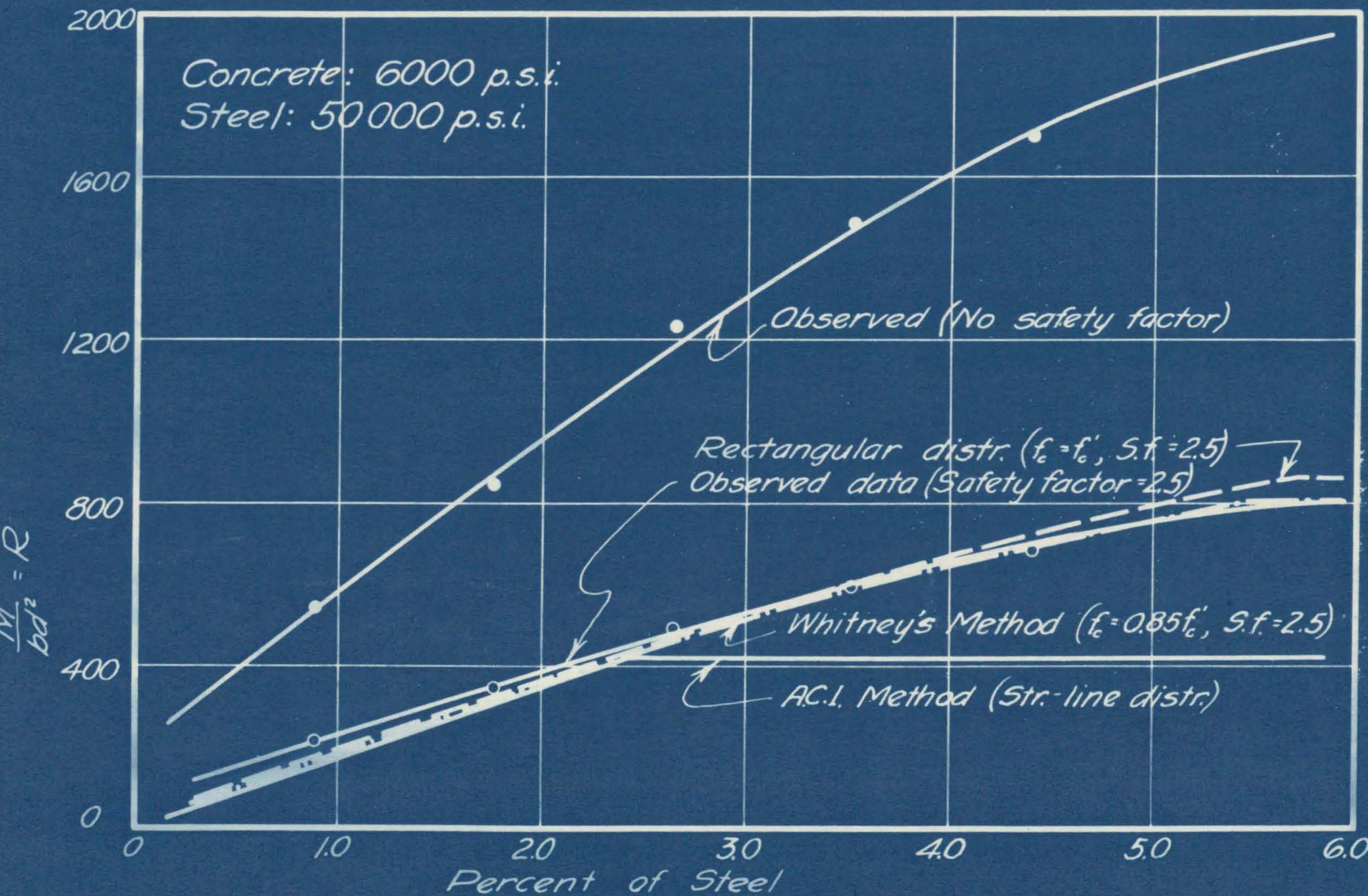


FIG. 48 - A COMPARISON OF OBSERVED AND COMPUTED "R" VALUES.

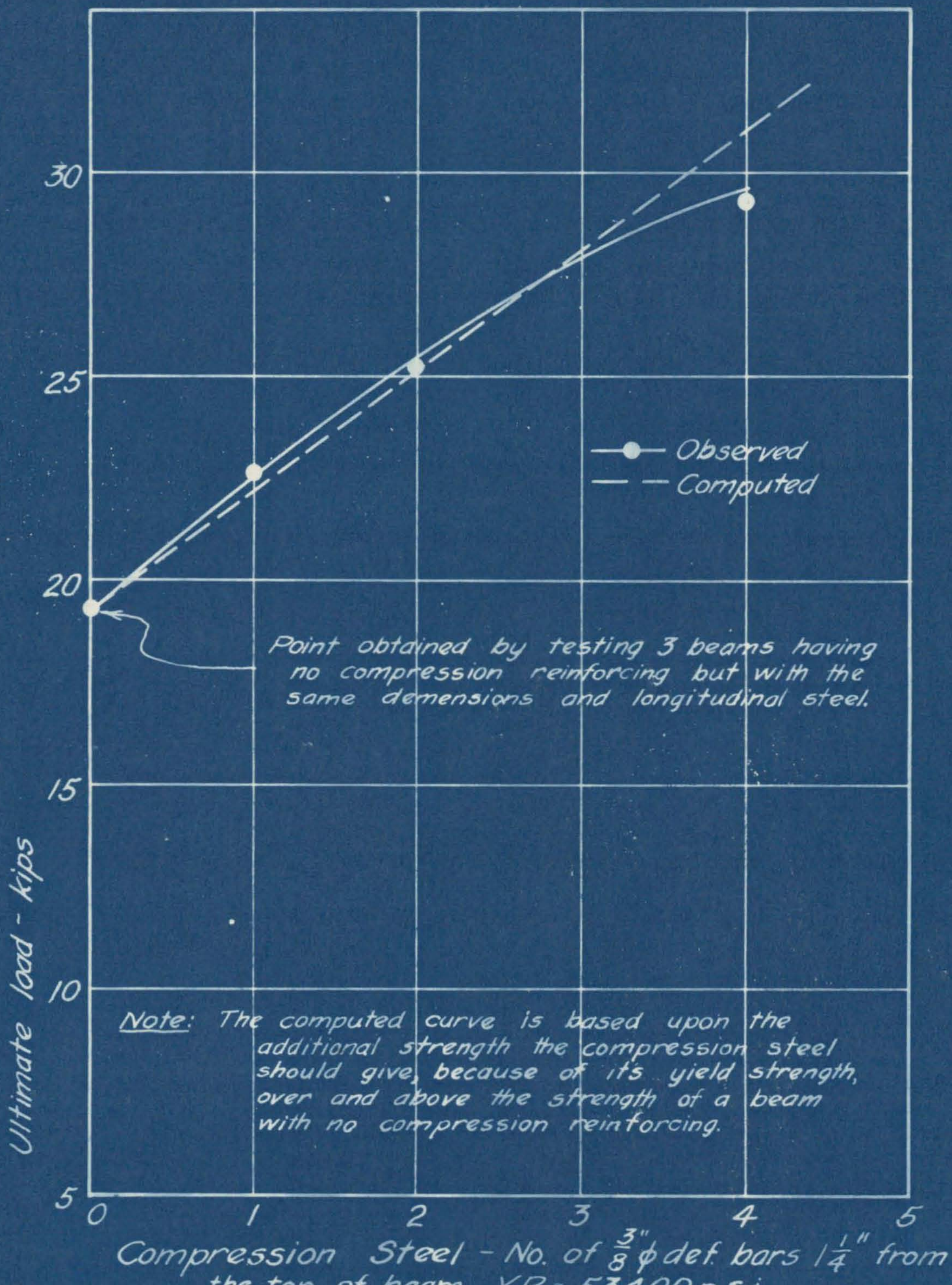


FIG. 49 - CURVE ILLUSTRATING THE EFFECTIVENESS OF COMPRESSION REINFORCING

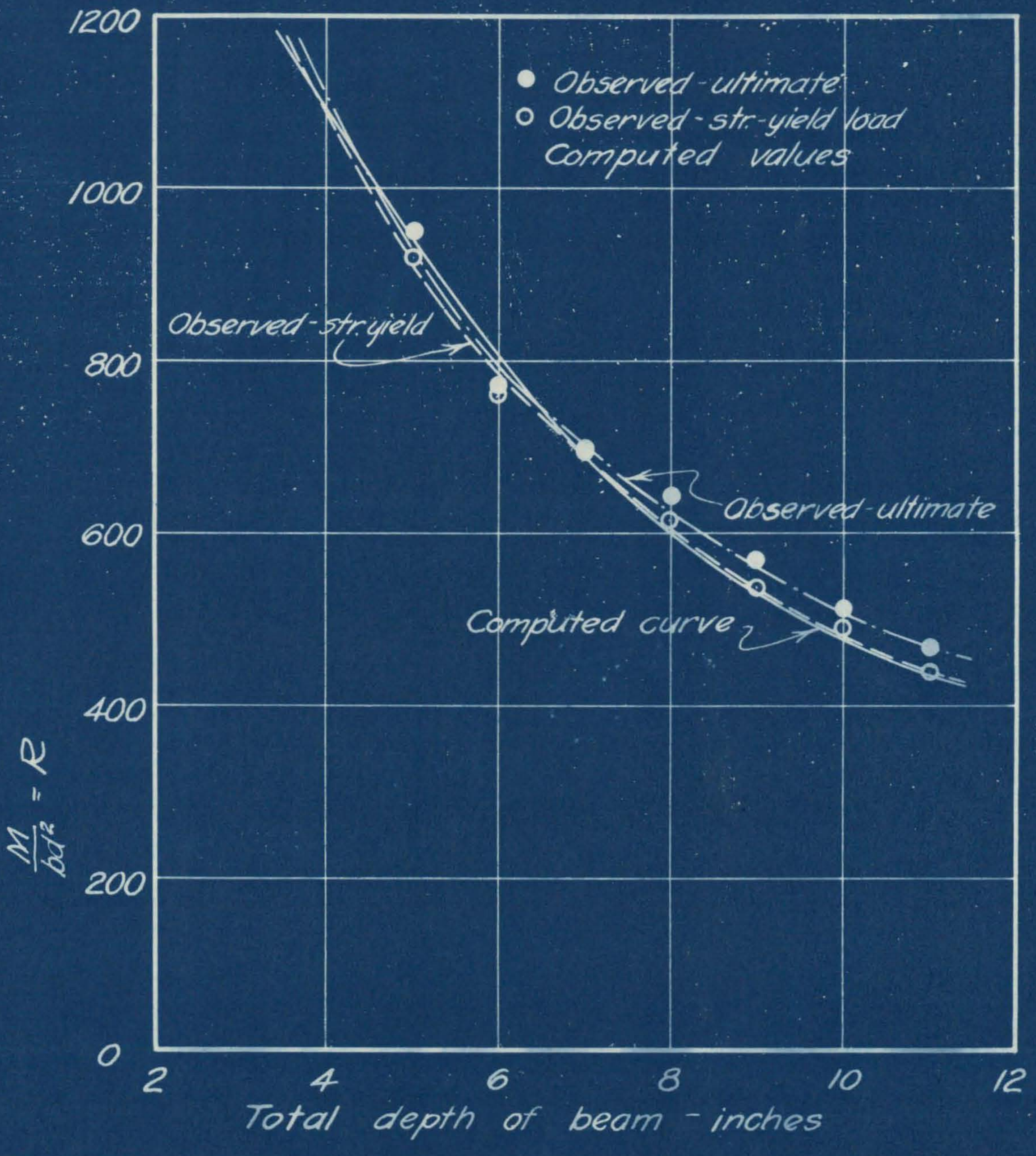


FIG. 50 - A COMPARISON OF OBSERVED AND COMPUTED "R" VALUES FOR BEAMS OF VARIOUS DEPTHS

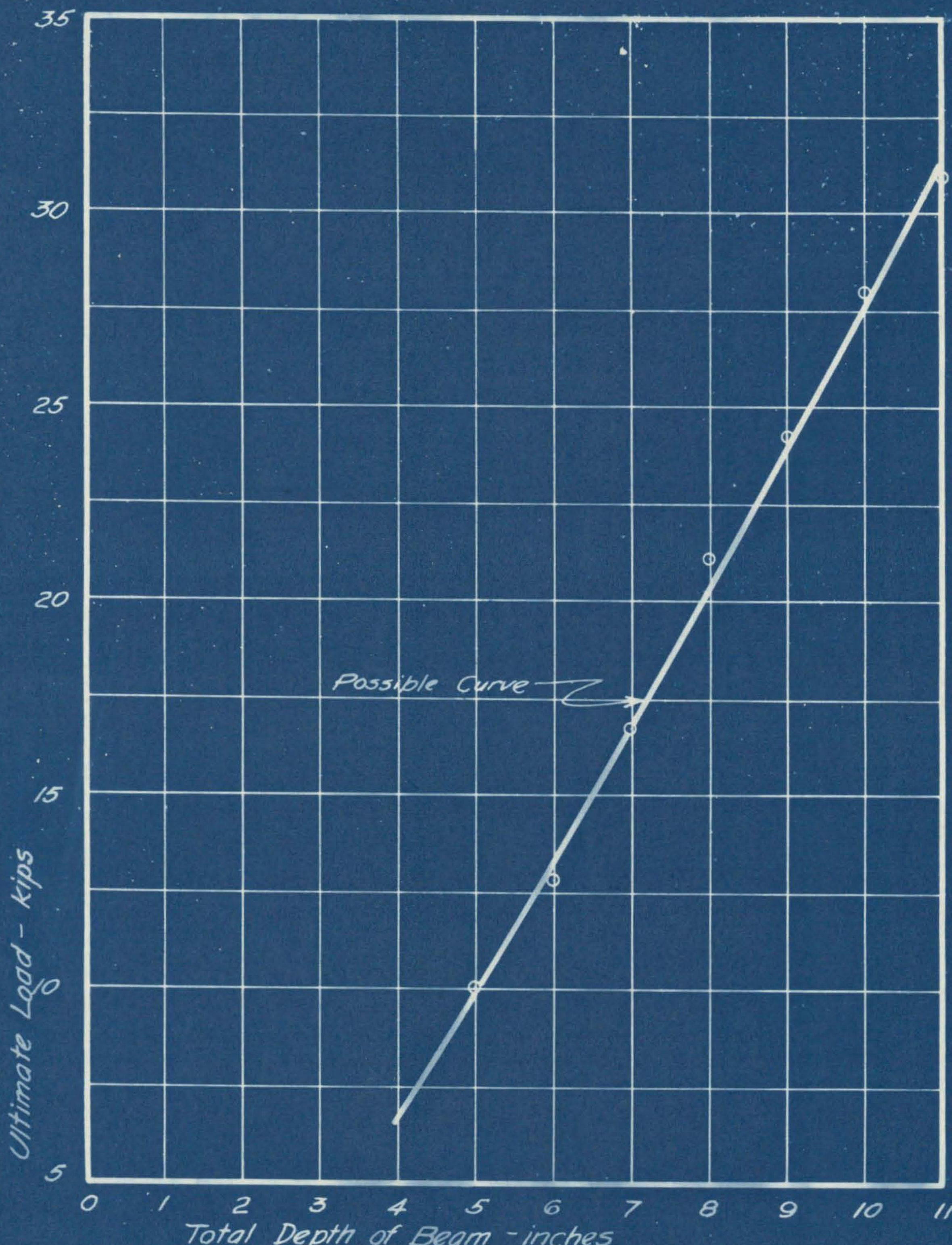


FIG. 51 - CURVE SHOWING THE ULTIMATE LOAD OF BEAMS with VARIOUS DEPTHS

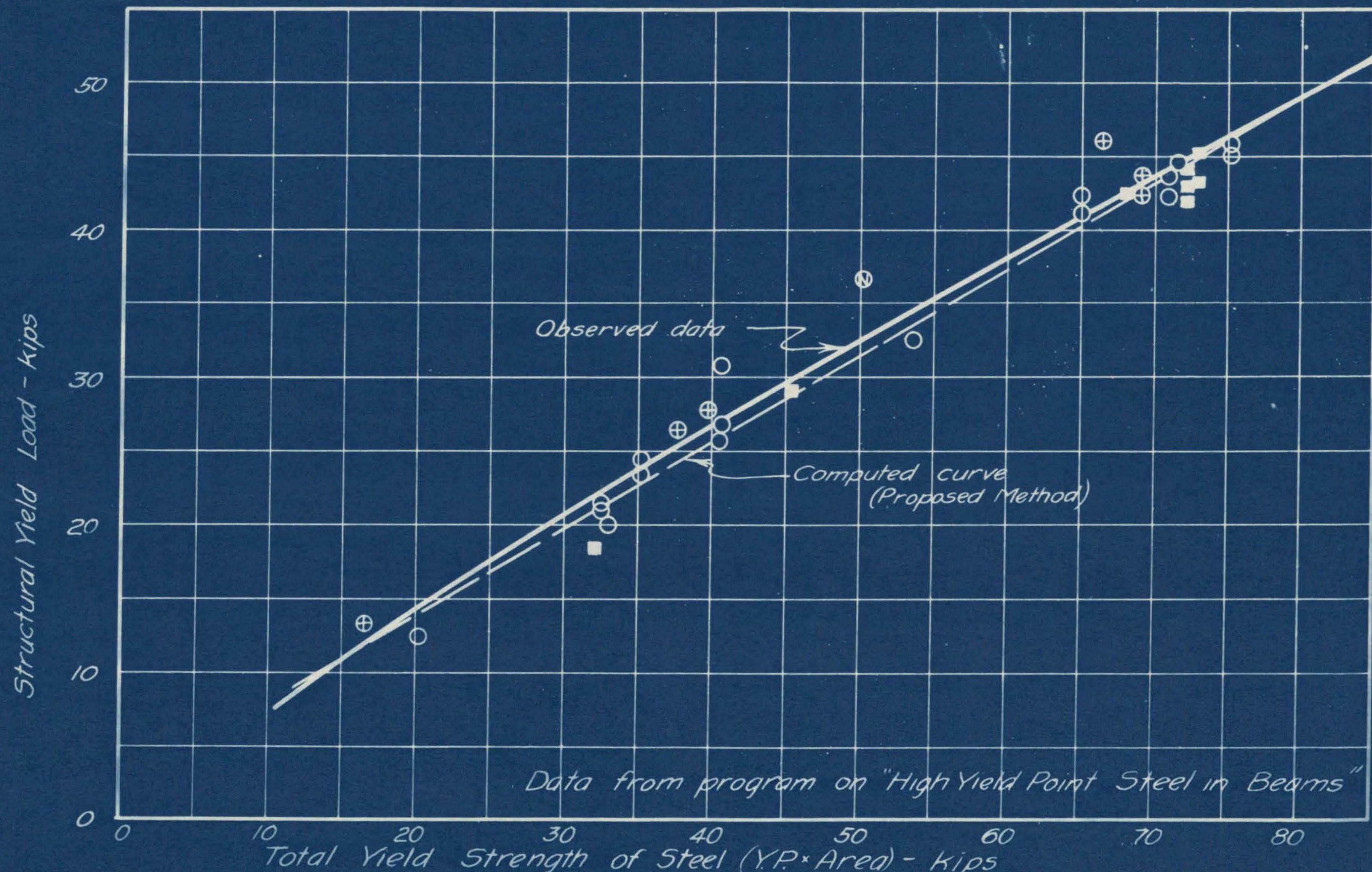


FIG. 52. - RELATION BETWEEN STRUCTURAL YIELD AND TOTAL YIELD STRENGTH OF REINFORCING STEEL

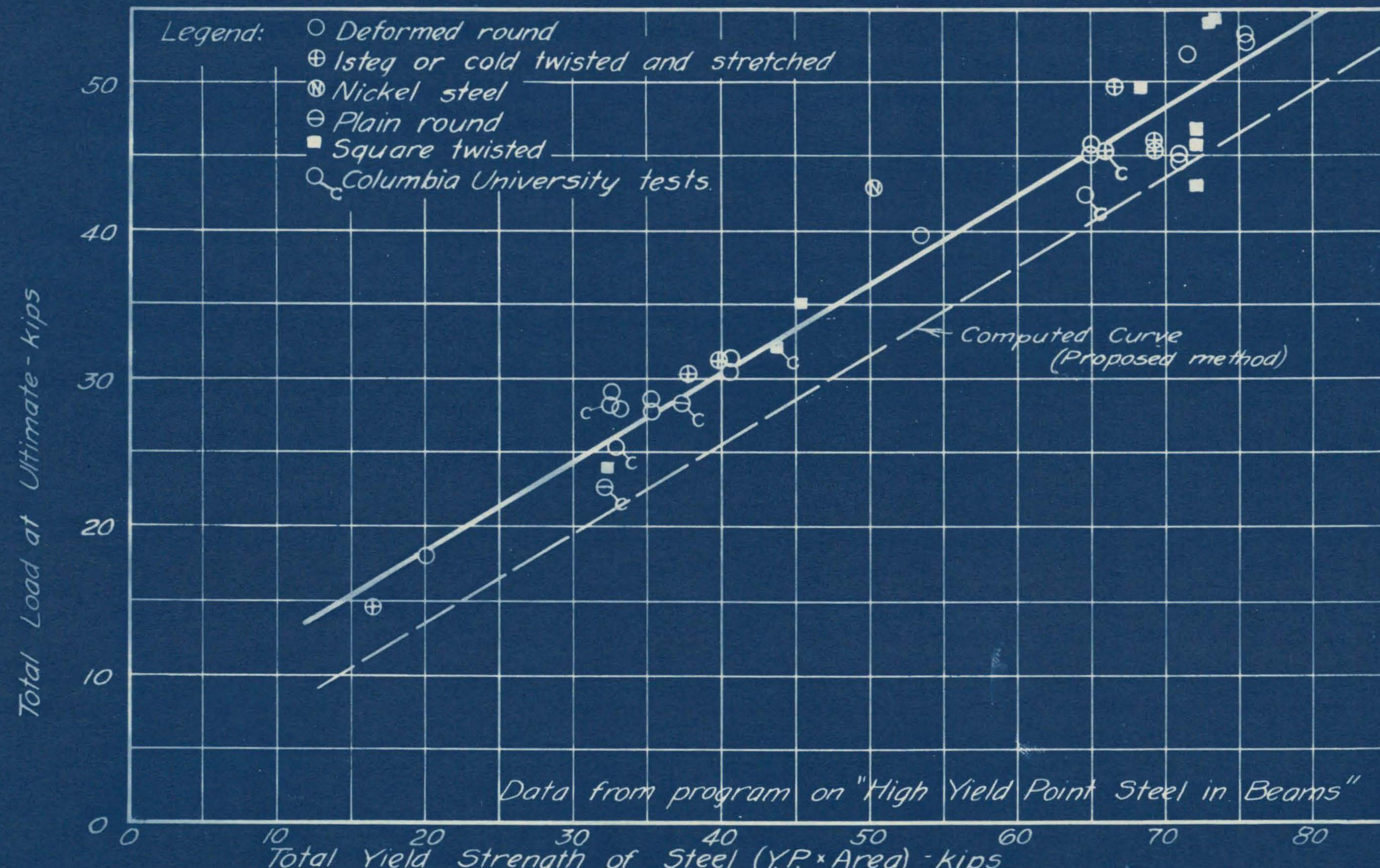


FIG. 53 - RELATION BETWEEN ULTIMATE STRENGTH OF BEAMS AND TOTAL YIELD STRENGTH OF REINFORCING STEEL

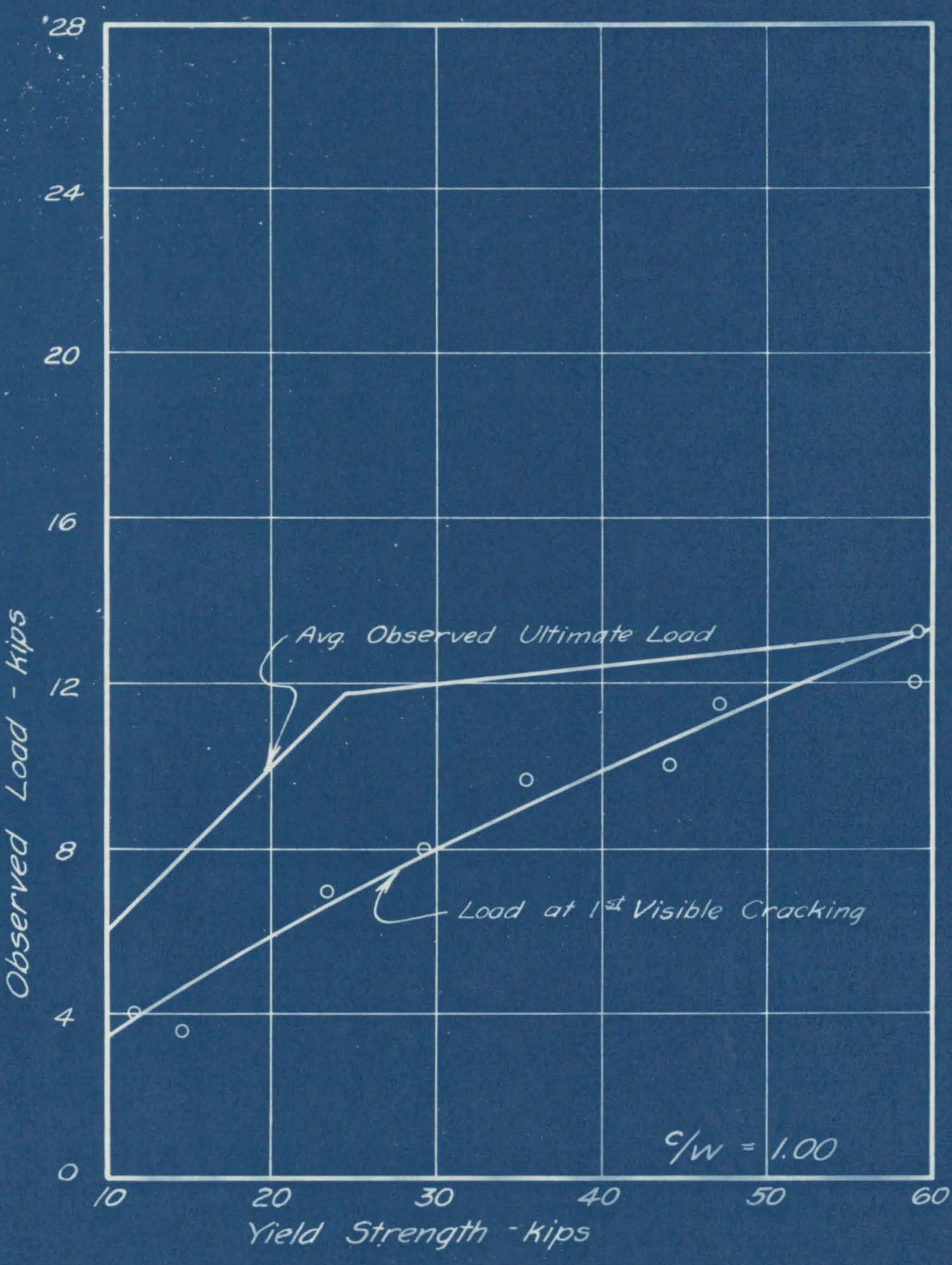


FIG. 54 - CURVE COMPARING ULTIMATE LOAD WITH THE LOAD AT 1st CRACKING

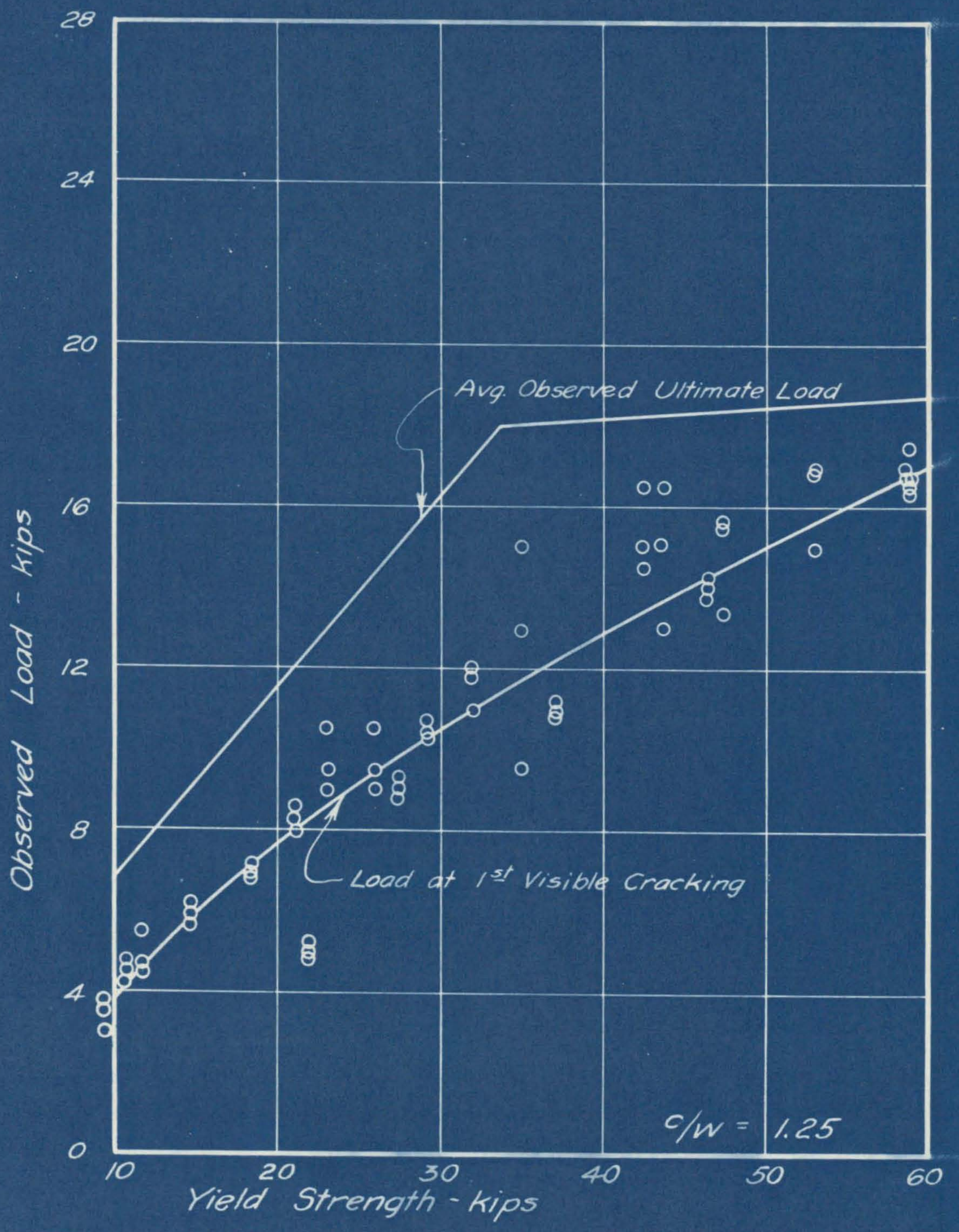


FIG. 55 - CURVE COMPARING ULTIMATE LOAD WITH THE LOAD AT 1st CRACKING

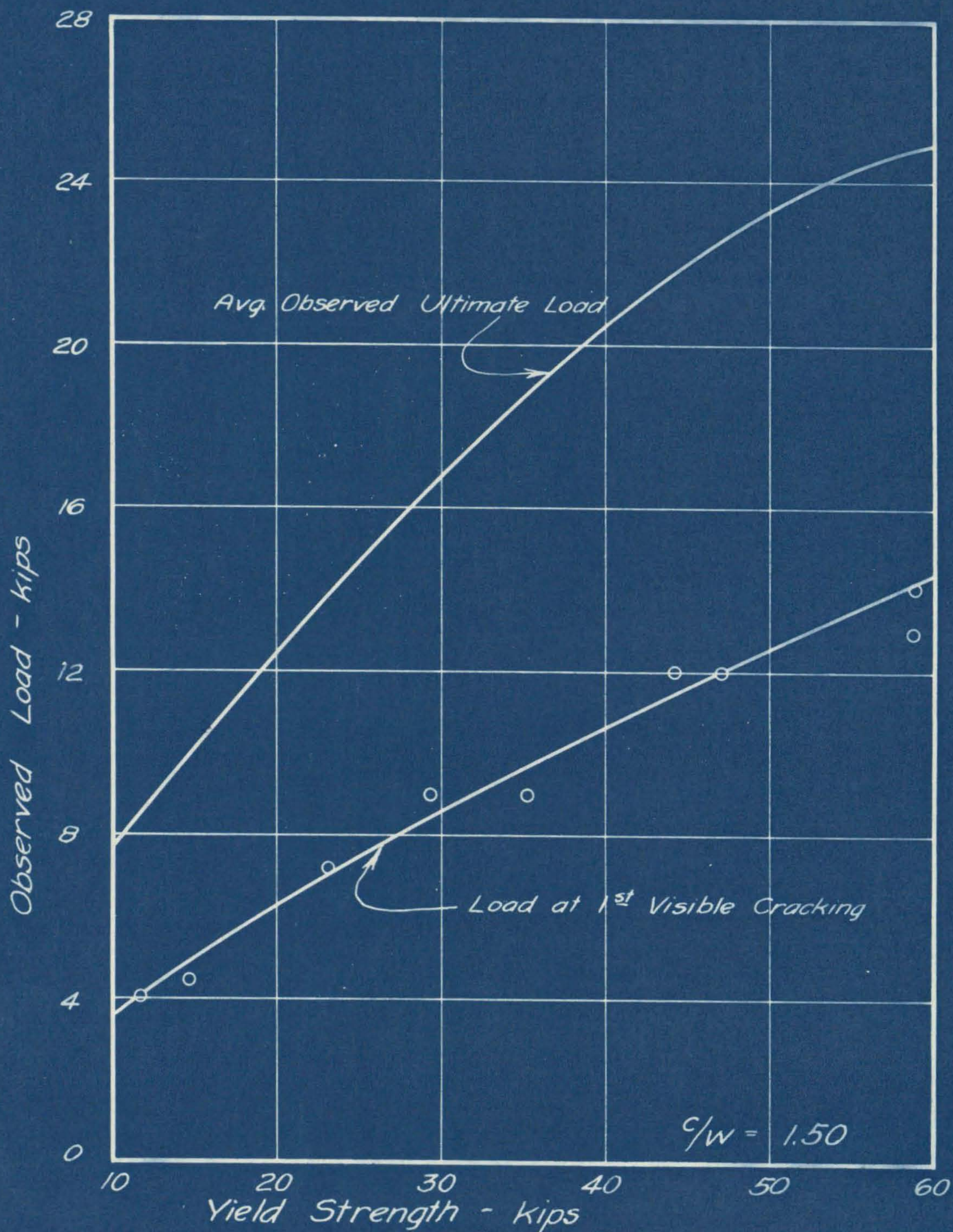


FIG. 56 - CURVE COMPARING ULTIMATE LOAD WITH THE LOAD AT 1ST CRACKING

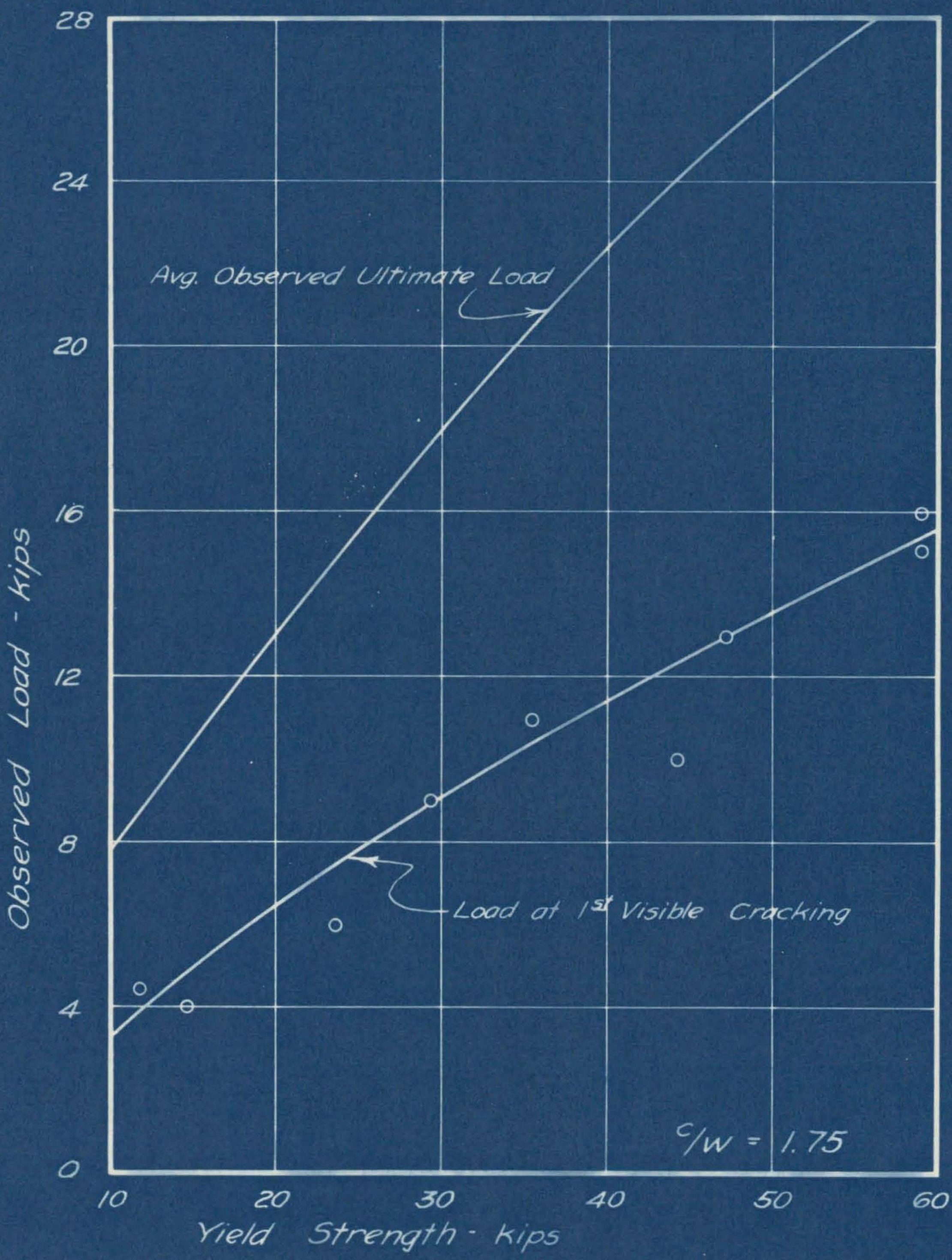


FIG. 57 - CURVE COMPARING ULTIMATE LOAD WITH THE LOAD AT 1st CRACKING

An Investigation
of Square Reinforced Concrete Slabs
Supported on All Sides

By

Otakar Ondra
Lehigh University
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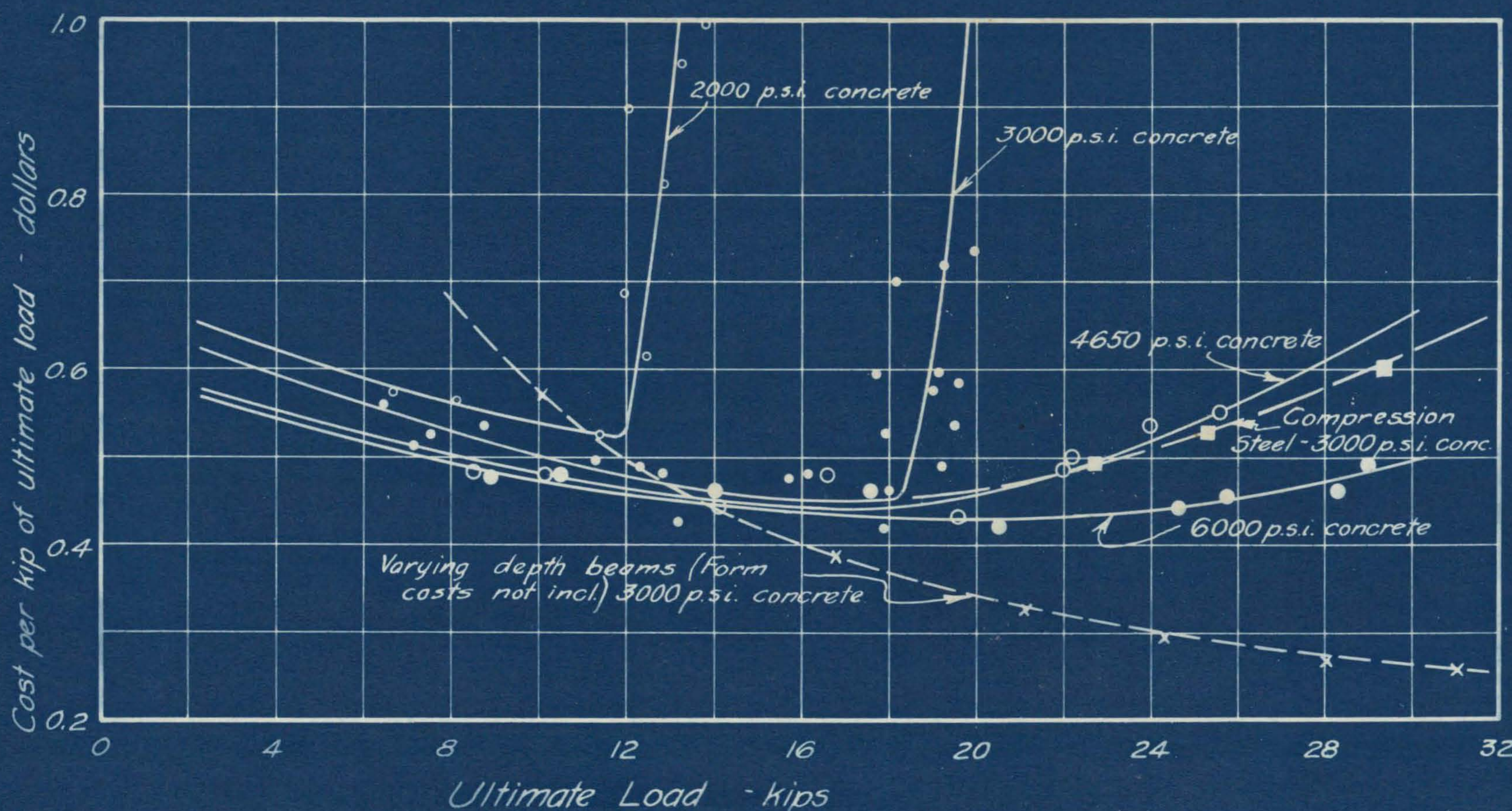

FIG. 58 - CURVE SHOWING RELATIVE COST PER KIP OF 1000 LINEAL INCHES OF SECTION